Exclusive Dealing and Entry by Competing Two-Sided Platforms*

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Abstract

We study competition between horizontally differentiated platforms offering exclusive and non-exclusive contracts to one side of the market (content providers). The introduction of non-exclusive contracts in addition to exclusive contracts softens the competition for content providers between platforms, as they have more tools to extract content providers' surplus. Users on the other side (consumers) pay the same price that they would pay in the exclusive-only game but gain access to new content provided by multi-homing content providers, increasing their surplus. Multi-homing content providers' surplus increases, while it decreases for those who single-home. Platforms charge more to exclusive and non-exclusive content providers, increasing their profits. As platform competition increases, platforms' profits increase if the network benefit on the consumers' side is high, so that the price effect dominates the market share effect. Finally, we show that platforms cannot deter entry by offering exclusive–non-exclusive contracts. In certain circumstances, platforms will jointly deviate from the exclusive–non-exclusive game and offer only exclusive prices in order to deter entry.

Keywords: Two-sided markets, platform price competition, network externalities, exclusive contracts, multi-homing

JEL Codes: D43, L4, L13, L14

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1 Introduction

Platforms offer exclusive and non-exclusive content. *Netflix*, *Hulu*, and *Amazon* are the major platforms in the video streaming business that purchase or license content rights and sell access to consumers. These platforms advertise their exclusive content to attract consumers; for example, *Netflix* advertises exclusive titles such as *Stranger Things* and *House of Cards*. However, consumers also value the myriad of easily accessible movie titles and sitcom series that are often available on multiple rival platforms (see Table B1 in Appendix B). The video game industry provides another example of platforms that serve both exclusive and non-exclusive content to consumers (see Table B2 in Appendix B). Eighth-generation video game consoles (PlayStation 4, Nintendo Switch, Xbox One) all have exclusive titles,[1] such as *Legend of Zelda* on Nintendo Switch, *Kingdom Hearts* on Sony PlayStation4, and *Halo* on Xbox One. Consumers value not only exclusive titles, but also non-exclusive titles, such as the *Lego* and *FIFA* series.[2]

How does the availability of both exclusive and non-exclusive contracts impact two-sided market outcomes? What determines the share of exclusive and non-exclusive content available on platforms? The answer to these questions has eluded previous studies due to the tipping feature of two-sided markets.[3] This paper seeks to answer these questions. We consider a model with n symmetric platforms serving a two-sided market with two types of users: *consumers* and *content providers*. Platforms are horizontally differentiated and offer content providers exclusive or non-exclusive contracts. If a content provider elects exclusivity, it pays the exclusive price and is contractually prohibited from joining rival platforms; if the content provider elects non-exclusivity, it joins all the other platforms by paying each platform's non-exclusive price. We assume that consumers single-home, or join a single platform exclusively. We study the subgame perfect Nash equilibrium of the two-stage problem, in which in stage 1, platforms simultaneously choose prices for both sides of the market, and in stage 2, users join the platforms, tak-

[1] A title is considered "exclusive" if it is not available on other game consoles. We do not consider its availability on PC.

[2] Examples of movie production companies include *20th Century Fox*, *Sony Pictures*, and *Warner Bros.*; examples of video game developers include *Electronic Arts*, *Blizzard*, and *Rockstar Games*. These content providers are not subsidiaries of any platform; they sell or license the rights to their content exclusively to one platform or non-exclusively to multiple platforms. We recognize the need to differentiate ownership of the content-production process, but we do not consider this particular problem (see, for example, Hagiu and Lee, 2011).

[3] The tipping feature is a result of the two-sided market feedback mechanism. If there is a small increase of side 1 users joining the platform, more side 2 users join the platform, which in turn attracts even more side 1 users. The feedback effect potentially cascades so that all users join the same platform despite idiosyncratic preferences. Current theoretical literature on exclusive and non-exclusive contracts such as Armstrong and Wright (2007) and Hagiu and Lee (2011), recognizes the tipping feature, and studies the cases where users fully unravel and join the same platform. These studies provide sufficient conditions for the existence of users choosing the non-exclusive contracts.

ing prices as given. We show that, under certain conditions, a subgame perfect Nash equilibrium exists.

We provide two examples of a unique symmetric subgame perfect Nash equilibrium with three platforms, in which the idiosyncratic preferences are exponentially and uniformly distributed (the two-platform game is presented in Appendix C and D). We show that the two models provide qualitatively similar results. That is, the introduction of non-exclusive contracts softens platforms' competition for content providers, as the platforms have a new margin to extract content providers' surplus—the exclusive–non-exclusive contract—in addition to the exclusive-exclusive margin. Consumers pay the same price as in the exclusive-only game, but gain access to new content provided by multi-homing content providers, increasing their surplus. Multi-homing content providers' surplus increases, but single-homing content providers' surplus decreases. Intuitively, content providers that remain single-homers are being charged more while gaining access to the same mass of consumers, resulting in a loss. Content providers that become multi-homers pay a price to each platform but receive multiple realizations of positive idiosyncratic preferences. Platforms' profits increase by two means: the higher price being charged to exclusive content providers, and the gains from charging more to non-exclusive content providers than to exclusive content providers. Finally, we show that welfare increases when the share of multi-homing content providers is small or close to one.

We show that when there are three platforms in the market in the exclusive–non-exclusive game, equilibrium prices are found to be higher than in the two-platform model. As the number of platforms increases, exclusive content providers are worse off due to the higher price and the smaller market share, whereas non-exclusive content providers are worse off if the discounted base utility is smaller than consumers' marginal network benefit. Although prices are higher, market shares are lower since there are more platforms in the market; platforms' profits increase if the network benefit on the consumers' side is high, so that the price effects dominate the market share effect. Thus, having more competing platforms in the market does not necessarily enhance consumer welfare in the exclusive–non-exclusive game.[4]

Finally, we explore the possibility of introducing or removing non-exclusive contracts as a way to deter entry. We consider a market with two incumbent platforms that can jointly commit to offer exclusive-only or exclusive–non-exclusive contracts to deter entry, and the entrant pays a fixed entry cost to enter the market. We show that platforms

[4] A similar results is found in the exclusive-only game: the overall welfare (i.e., the sum of consumers' surplus, content providers' surplus, and platforms' profit) decreases when the number of platforms increases from two to three (see, Tan and Zhou, 2020). Similarly, Bryan and Gans (2019) showed that a monopoly platform is welfare superior to duopoly platforms by using a tipping model, where all users multi-home à la Armstrong and Wright (2007). Our paper complements Bryan and Gans (2019).

cannot deter entry by offering exclusive–non-exclusive contracts. However, in certain circumstances, platforms will jointly deviate from the exclusive–non-exclusive game and offer exclusive prices only in order to deter entry.

In two-sided markets, increases in the market share on one side of the market attracts users from the other side, which attracts even more users from the original side. Ellison and Fudenberg (2003) explore conditions in two-sided market models that can lead to a knife-edge equilibrium (or tipped corner solutions) or a plateau of quasi-equilibria—many stable equilibria with unequal market sizes. To circumvent the tipping problem, Carrillo and Tan (2018) and Choi (2010) opt to exogenously split users into exclusive or non-exclusive types, which ensures that the weaker platform is not necessarily foreclosed as a result of tipping. Athey et al. (2016) use an exogenous type model to study the impact of multi-homing users on content production.

An alternative is to assume that platform differentiation on one side of the market (e.g., sellers) is so small that all sellers multi-home. Armstrong and Wright (2007) focus on analyzing the equilibrium of a two-sided market model where either all users on one side tip to one platform, and the equilibrium where all users on one side multi-home to both platforms. In contrast, we assume that platform differentiation is not too small, so that multi-homers and single-homers can coexist in equilibrium, similar to Belleflamme and Peitz (2019) and Liu et al. (2019). Belleflamme and Peitz (2019) study a two-platform game where one side of the market (e.g., sellers) is allowed to multi-home. Sellers (content providers in our model) choose between single-homing or multi-homing to all platforms, while buyers (consumers in our model) single-home.[5] They find conditions under which multi-homing is welfare-enhancing (i.e., platforms' profits, buyer surpluses, and seller surpluses increase). Under certain conditions, we find that this result holds for two and three platforms. Liu et al. (2019) study a two-sided platform game in which both buyers and sellers can multi-home, and platforms compete on transaction fees charged on both sides. They show that the effect of increasing platform competition depends on whether buyers multi-home or not; platform competition shifts the fee structure in favor of buyers if buyers are single-homing but shifts the fee structure in favor of sellers if buyers are multi-homing.[6] We complement these studies by offering a mathematically

[5]Our paper extends Belleflamme and Peitz (2019) in three ways. First, our model allows for non-correlated idiosyncratic preferences (e.g., exponential and uniform distributions). Second, we consider more than two platforms. Our model is more tractable when the number of platforms is larger than two. Third, we distinguish between exclusive and non-exclusive prices for content providers. In Belleflamme and Peitz (2019), content providers that opt to multi-home pay the sum of each platform's exclusive price. Distinguishing between exclusive and non-exclusive prices allows us to study exclusive discounts that platforms offer to attract exclusive content providers. For a discussion of why platforms are interested in offering discounts to exclusive content providers, see Section 6.

[6]The model suggested by Liu et al. (2019) differs from our model in many aspects. First, in their model, platforms are differentiated from the buyer's perspective but identical from the seller's perspective,

tractable model that allows us to identify equilibria in which platforms offer exclusive and non-exclusive contracts.

Our random utility maximization model is similar to Tan and Zhou (2020), who study a game in which platforms offer exclusive contracts only to content providers, whereas we consider a game in which platforms offer non-exclusive prices in addition to exclusive prices. We complement Tan and Zhou (2020) by studying the exclusive–non-exclusive game with n-platforms and general market shares.

Note that our paper provides an intuition as to why platforms do not have incentives to offer exclusive contracts similar to the previous literature on one-sided models with horizontally differentiated consumers (Bernheim and Whinston, 1998; Calzolari and Denicolò, 2013; Mathewson and Winter, 1987; O'Brien and Shaffer, 1997).[7] Exclusive contracts intensify competition, leading to lower prices and profits. If non-exclusive prices are also offered, exclusive prices are higher, as platforms have more tools to extract content surplus. Thus, non-exclusive prices soften competition.

Previous empirical literature has studied the impact of exclusive contracts on platform choice. Corts and Lederman (2009) and Nair (2007) study the impact of exclusive and non-exclusive titles on console adoption. Corts and Lederman (2009) report that 40% of total video game titles are non-exclusively available on multiple consoles and Prieger and Hu (2008) find that "PlayStation 2 and Xbox garner most of their revenue from non-exclusive titles." While certain titles are produced by the platforms themselves, most of the content available on platforms is produced by third-party movie producers and video game developers. Lee (2013) estimates the impact of exclusive titles on the adoption of sixth-generation video game consoles, concluding that exclusivity promotes the entry of smaller platforms but does not benefit the larger incumbents. These studies further motivate the need for a theoretical approach to understand how exclusive and non-exclusive contracts impact content availability.

Our paper also contributes to the literature on competitive price discrimination (e.g., Armstrong and Vickers, 2001, 2010; Chica and Tamayo, 2020; Hoernig and Valletti, 2007;

while in our model, platforms are differentiated from both buyer's and seller's perspectives. Second, we distinguish between exclusive and non-exclusive prices for content providers. In the model used by Liu et al. (2019), sellers pay the same price—regardless of their homing decision—to each platform they join. Finally, the utility and profit functional forms of both models are different.

[7] Calzolari and Denicolò (2013) considers a one-sided model in which consumers are horizontally differentiated and firms compete by offering exclusive and non-exclusive non-linear contracts. The authors show that when firms compete in non-linear pricing, the competition is limited by the extent of product differentiation. However, when exclusive contracts are introduced, firms compete in utility space, which intensifies competition, resulting in lower prices and profits. Mathewson and Winter (1987) find that firms do have incentives to offer exclusive contracts, which may have anticompetitive effects. However, O'Brien and Shaffer (1997) and Bernheim and Whinston (1998) show that these results depend on firms being restricted to linear pricing (i.e., two-part tariffs [or nonlinear tariffs] restore the neutrality of the exclusive contracts).

Rochet and Stole, 2002; Tamayo and Tan, 2020; Yang and Ye, 2008). Most of the previous literature on competitive price discrimination considers only one-sided models but finds a similar intuition: When the number of tools available to firms (platforms in our case) increases, firms can extract consumers' (or content providers') surplus more efficiently.

Finally, our results have antitrust implications regarding exclusive contracts and platform ownership.[8] First, we show that exclusive contracts in certain cases can be helpful in deterring entry. Our findings differs from Lee (2013), who shows that exclusive contracts promote platform entry. Second, we show that having more competing platforms in the market does not necessarily enhance consumer welfare.

The paper proceeds as follows. Section 2 sets up the model. Section 3 solves the exclusive–non-exclusive game via backward induction. Section 4 provides two examples in which a unique symmetric subgame perfect Nash equilibrium exists. Section 5 examines the impact of platform entry on consumers and content providers' prices, platforms' profits and welfare in the exclusive–non-exclusive game. Section 6 discusses different real life examples of two-sided platforms and how they fit our model prediction. Section 7 concludes.

2 Model

Consider a model with n symmetric platforms serving a two-sided market. There are two types of users, one type on either side of the market. Side 1 users are *consumers*, and side 2 users are *content providers*. We study the subgame perfect Nash equilibrium of the two-stage problem: In stage 1, platforms simultaneously choose prices for both sides of the market; in stage 2, users join the platforms taking prices as given.

We study an exclusive–non-exclusive game where platforms offer content providers a choice between exclusivity and non-exclusivity. If a content provider elects exclusivity, it pays the exclusive price and is contractually prohibited from joining rival platforms. If the content provider elects non-exclusivity, it joins all the other platforms by paying each platform's non-exclusive price. Furthermore, we assume that consumers single-home (join a single platform exclusively) and full market coverage—consumers and content providers join at least one platform.

We raise two questions on how to model multi-homing users. First, do they receive a

[8]Note that our results contrast with the anticompetitive effects studied in the exclusive contract literature in a variety of settings; namely, contracts signed between the buyers and incumbent that may block the entry of a (more efficient) firm (Aghion and Bolton, 1987; Rasmusen et al., 1991; Segal and Whinston, 2000), and when buyers compete in downstream markets (Asker and Bar-Isaac, 2014; Fumagalli and Motta, 2006; Johnson, 2012; Simpson and Wickelgren, 2007; Wright, 2009). We show that non-neutrality of the exclusive contracts relies on the ability of the platforms to extract content providers rent and the network effects.

new draw of an independently and identically distributed preference or the sum of preferences for individual platforms? We chose the latter for its natural interpretation. Models using discrete choice specification can simplify the system by assuming the multi-homing option receives a *new draw* of preference independent from single-homing preferences. We favor the add-up specification for its intuitive interpretation of the combination being the sum of its parts.[9] Also, choosing the add-up specification ensures that content providers do not make homing decisions that are inconsistent with their idiosyncratic preferences for individual platforms.

Second, when there are n platforms, do users multi-home to all platforms or multi-home to any subsets of platforms? Similar to Liu et al. (2019), we assumed that multi-homers join all platforms in the market.[10] We assume that multi-homing content providers join all platforms for two reasons.[11] First, it allows us to simplify the problem of the content provider; instead of modeling all the possible choices where it can multi-home, we need to consider only two ex-post types of content providers in the general n-platform case—those that single-home and those that multi-home to all n platforms. Second, multi-homers joining all platforms is an implied result from models with non-negative idiosyncratic terms and non-positive equilibrium non-exclusive prices. When there are n platforms, a content provider that already decided to join two platforms necessarily joins the third platform, since the content provider receives a payment from each platform it joins and realizes additional idiosyncratic preferences. Hence, joining all platforms strictly dominates multi-homing to less than all platforms.[12]

Our model is solved via backward induction. We first present stage 2 users' problem

[9] For example, if a video producer has a certain level of idiosyncratic preference for *Netflix* and *Hulu* as distribution channels, then the utility of multi-homing should intuitively be the sum of given idiosyncratic values (possibly with additional noise). If the video producer values the combination of *Netflix* and *Hulu* with a new draw of idiosyncratic value, it would allow for the possibility that a video producer who highly favors either platform individually might value the combination of both platforms below that of an individual platform.

[10] There is an inherent asymmetry between the users on both sides regarding their homing decision: one side can multi-home but the other side always single-homes. Examples include video streaming platforms (such as *Netflix*, *Amazon Prime Video*, and *Hulu*) and game-publishing platforms (such as *Steam* and *Google Play Store*). These markets are characterized by having many content providers and contractible exclusive contracts. Our model is not compatible with markets such as dating platforms, where users are symmetric across sides and exclusive contracts are unenforceable. We analyze platforms' incentive to offer exclusive discounts to content providers and how it impacts content providers' decisions to join platforms exclusively or non-exclusively.

[11] Liu et al. (2019) study a two-sided platform game in which both buyers and sellers can multi-home and platforms compete on transaction fees charged to both sides. They show that, depending on the seller's per-transaction surplus, the seller: (i) joins no platform, (ii) joins all platforms $j \neq i$ except platform i, or (iii) joins all platforms.

[12] Our model does not rely on any assumptions on the sign of prices offered by platforms. Live-streaming platforms are an example of this type of market—the cost of publishing to additional platforms is low, content providers are compensated for their broadcasts, and content providers either single-home or multi-home to all platforms.

of joining platforms, then stage 1 platforms' problem of choosing prices. In stage 2, users take platforms' prices as given. Each consumer joins the platform that offers the highest utility; each content provider chooses between joining a platform exclusively or multi-homing to multiple platforms non-exclusively.

Consumer utility from exclusively joining platform $i \in \mathcal{N} = \{1, 2, ..., n\}$ is

$$\tilde{u}_1^i = v_1 + \phi_1 \left(n_2^i, n_2^m\right) - p_1^i + \varepsilon_1^i, \tag{1}$$

where v_1 is a base utility common to all platforms, n_2^i is the mass of exclusive content providers available on platform i, and n_2^m is the mass of non-exclusive content providers available on all platforms. The mapping ϕ_1 is a function from $\mathbb{R}^2$ to $\mathbb{R}$, and $\phi_1 (n_2^i, n_2^m)$ measures the network benefit for consumers derived from content available on platform i. p_1^i is the lump-sum fee that platform i charges to each consumer for accessing its available content, and ε_1^i is the idiosyncratic preference for platform i of each consumer, which is a random variable independently and identically distributed (i.i.d.) across consumers, with cumulative distribution function (c.d.f.) F_1, and an associated probability density function (p.d.f.) f_1. We assume F_1 is continuous and differentiable everywhere. Let $u_1^i \equiv v_1 + \phi_1 (n_2^i, n_2^m) - p_1^i$ be the deterministic component of the utility.

Consumers who join platform i are those who prefer single-homing to platform i than other platform. The mass of consumers single-homing to platform i is

$$n_1^i = \Pr\left(\tilde{u}_1^i > \max_{j \neq i} \tilde{u}_1^j\right). \tag{2}$$

Content provider utility from exclusively joining platform $i \in \mathcal{N}$ is

$$\tilde{u}_2^i = v_2 + \phi_2 \left(n_1^i\right) - p_2^i + \varepsilon_2^i. \tag{3}$$

The interpretation of each term is analogous to side 1 consumer utility. Since consumers do not multi-home, content providers derive network benefit only from single-homing consumers on platform i, n_1^i. Each content provider has an idiosyncratic preference ε_2^i for platform i, which is a random variable i.i.d. across content providers with c.d.f. F_2 and associated p.d.f. f_2. Similarly, we assume F_2 is continuous and differentiable everywhere. Let $u_2^i \equiv v_2 + \phi_2 (n_1^i) - p_2^i$ be the deterministic component of the utility from exclusively joining platform i.

Content provider utility from non-exclusively joining all platforms is

$$\tilde{u}_2^m = v_2 \left(1 + (n-1)\delta\right) + \phi_m \left(n_1^1, \ldots, n_1^n\right) - \sum_{i \in \mathcal{N}} p_{2m}^i + \sum_{i \in \mathcal{N}} \varepsilon_2^i. \tag{4}$$

When a content provider multi-homes, it receives base utility v_2 from the first platform, but only a discounted base utility, δv_2, for each additional platform. We assume that $\delta < 1$, which reflects the loss in utility stemming from duplicated services provided by multiple platforms. A non-exclusive multi-homing content provider derives utility from the mass of consumer available on the platforms it joins, in return it must pay the fee p^i_{2m} to each platform $i \in \mathcal{N}$. Let

$$u^m_2 \equiv \underbrace{v_2 \left(1 + (n-1)\delta\right)}_{\equiv \tilde{v}_2} + \phi_m \left(n^1_1, \ldots, n^n_1\right) - \sum_{i \in \mathcal{N}} p^i_{2m}$$

be the deterministic component of the utility from non-exclusively joining all platforms. Each platform offers content providers a non-exclusive price in addition to the exclusive price. Content providers who join platform i are those who prefer single-homing to platform i than single-homing to any other platform or multi-homing to all platforms. The mass of content providers single-homing to platform i is

$$n^i_2 = \Pr\left(\tilde{u}^i_2 > \max_{j \neq i}\{\tilde{u}^j_2, \tilde{u}^m_2\}\right). \tag{5}$$

Content providers who multi-home to join all platforms prefer multi-homing to any single-homing option. The mass of multi-homing content providers is

$$n^m_2 = \Pr\left(\tilde{u}^m_2 > \max_{i}\{\tilde{u}^i_2\}\right). \tag{6}$$

In stage 1, each platform i sets prices p^i_1, p^i_2 and p^i_{2m} to solve the problem of

$$\max_{\{p^i_1, p^i_2, p^i_{2m}\}} \pi^i\left(p^i_1, p^i_2, p^i_{2m}\right) = n^i_1 p^i_1 + n^i_2 p^i_2 + n^m_2 p^i_{2m}, \tag{7}$$

where n^i_1, n^i_2, and n^m_2 are defined by (2), (5), and (6), respectively. Note that multi-homing content providers pay a total price of $\sum_i p^i_{2m}$, which is set by platforms jointly but non-cooperatively. We refer to the difference between exclusive and non-exclusive prices, $p^i_{2m} - p^i_2$, as the *exclusive premium*—the incentive provided by the platform to convert a non-exclusive content provider into an exclusive content provider.

Our model generalizes the exclusive-price-only model studied by Tan and Zhou (2020), where both consumers and content providers are restricted to single-homing.[13] We use

[13] In this case, platform i sets exclusive prices p^i_1 and p^i_2 to solve the problem of

$$\max_{\{p^i_1, p^i_2\}} \pi^i(p^i_1, p^i_2) = n^i_1 p^i_1 + n^i_2 p^i_2, \tag{8}$$

where n^i_1 is defined by (2) and n^i_2 is analogously defined as n^i_1. Tan and Zhou (2020) study the model for the case of s (≥ 1) sides and n platforms.

the exclusive-only game as a benchmark model to compare against the exclusive–non-exclusive game.

3 The Symmetric Equilibrium

The symmetric equilibrium is solved via backward induction. First, stage 2 users choose which platform to join (or whether to join all platforms in the case of content providers) taking stage 1 prices as given; then, stage 1 platforms maximize benefits according to how users would react to different sets of prices. To derive our equilibrium solution, we follow a strategy similar to that of Tan and Zhou (2020), who study the exclusive-only game. In stage 2, given a vector of prices $\boldsymbol{P}$, we show that users uniquely allocate themselves among the platforms (i.e., the vector of market shares $\boldsymbol{n}$ is determined) so that the markets clear up. In our setting, content providers multi-home to all platforms. Thus, if a single-homing content provider deviates from the equilibrium, it has two options: either to single-home to a different platform or to multi-home to all the platforms. The fact that content providers can choose between single-homing or multi-homing imposes new challenges to guarantee that the mapping $\boldsymbol{n} \mapsto \boldsymbol{P}(\boldsymbol{n})$ is well defined. Second, we simplified our problem, reducing it to the existence and uniqueness of a single variable, β, that determines how attractive multi-homing is compared to single-homing for content providers, and show that a subgame perfect Nash equilibrium exists.

3.1 Stage 2

In stage 2, users take stage 1 prices as given and select the choice that maximizes their utility. Let $\boldsymbol{u} = (u^1, \cdots, u^n, u^m) \in \mathbb{R}^{n+1}$ be a vector of deterministic utilities from each of the $n+1$ options for users in the market. For $i \in \mathcal{N}$, and $k \in \mathcal{S} = \{1, 2\}$, the quantity $Q_k^i(\boldsymbol{u})$ measures how many users from side k join platform i given the utility levels in $\boldsymbol{u}$. $Q_1^i : \mathbb{R}^{n+1} \to [0, 1]$ is defined as

$$Q_1^i(\boldsymbol{u}) = Q_1^i\left(u^1, \ldots, u^n, u^m\right) \equiv \mathbb{P}\left(u^i + \varepsilon_1^i \geq \max_{j \neq i,\, j \in \mathcal{N}}\left(u^j + \varepsilon_1^j\right)\right). \tag{9}$$

For side 2, $Q_2^i : \mathbb{R}^{n+1} \to [0, 1]$ is defined as

$$Q_2^i(\boldsymbol{u}) = Q_2^i\left(u^1, \ldots, u^n, u^m\right) \equiv \mathbb{P}\left(u^i + \varepsilon_2^i \geq \max_{j \neq i,\, j \in \mathcal{N}}\left(u^j + \varepsilon_2^j\right), u^i + \varepsilon_2^i \geq u^m + \boldsymbol{\varepsilon}_2 \cdot \mathbf{1}_n\right), \tag{10}$$

where $\boldsymbol{\varepsilon}_2 \cdot \mathbf{1}_n = \sum_{i \in \mathcal{N}} \varepsilon_2^i$. We refer to $Q_k^i(\cdot)$ as the exclusive participation function for platform i on side k. The quantity $Q_2^m(\boldsymbol{u})$ measures the number of content providers that multi-home to all platforms given the utility levels in $\boldsymbol{u}$, and it is defined as $Q_2^m : \mathbb{R}^{n+1} \to [0,1]$,

$$Q_2^m(\boldsymbol{u}) = Q_2^i\left(u^1, \ldots, u^n, u^m\right) \equiv \mathbb{P}\left(u^m + \boldsymbol{\varepsilon}_2 \cdot \mathbf{1}_n \geq \max_{j\in\mathcal{N}}\left(u^j + \varepsilon_2^j\right)\right). \tag{11}$$

We refer to $Q_2^m(\cdot)$ as the non-exclusive participation function on side 2. Note that, given that consumers always single-home, it follows that $Q_1^m(\boldsymbol{u}) = 0$ for all $\boldsymbol{u} \in \mathbb{R}^{n+1}$.[14]

Let $\boldsymbol{P} = (p_k^i, p_{2m}^i)_{i\in\mathcal{N}}^{k\in\mathcal{S}}$ be a vector of prices, and let $\boldsymbol{n} = (n_k^i, n_2^m)_{i\in\mathcal{N}}^{k\in\mathcal{S}}$ be a vector of market shares. A *participation equilibrium* (PE) is a set of stable user choices based on prices and other users' actions. More precisely, the vector $\boldsymbol{n}$ is a PE associated with $\boldsymbol{P}$ if and only if it solves the following system of equations:

$$\begin{cases} n_k^i &= Q_k^i\left(\left(v_k + \phi_k\left(n_l^j, n_2^m\right) - p_k^j\right)_{j\in\mathcal{N}}, \tilde{v}_2 + \phi_m\left(\boldsymbol{n}_1\right) - \boldsymbol{p}_{2m}\cdot \mathbf{1}_n\right), \\ n_2^m &= Q_2^m\left(\left(v_k + \phi_k\left(n_l^j, n_2^m\right) - p_k^j\right)_{j\in\mathcal{N}}, \tilde{v}_2 + \phi_m\left(\boldsymbol{n}_1\right) - \boldsymbol{p}_{2m}\cdot \mathbf{1}_n\right), \end{cases} \tag{12}$$

for all $i, j \in \mathcal{N}$, $i \neq j$, and for all $k, l \in \mathcal{S}$, $k \neq l$. The vector of market shares $\boldsymbol{n} = \boldsymbol{n}(\boldsymbol{P})$ is implicitly determined by the system of equations (12).

Let $\overline{q}$ be the maximal upper bound of the slopes of the participation functions, and let $\overline{\Phi}$ be the maximal upper bound of the slopes of the network benefit functions. The constant $\overline{q}$ represents the sensitivity of the market share function to the utility offered by each platform, and $\overline{\Phi}$ represents the sensitivity of the marginal network benefit to content availability. Existence of a PE follows from *Brouwer's fixed-point theorem*; uniqueness follows from *Banach's fixed-point theorem*, provided that $\overline{q}\overline{\Phi}$ is small enough. We summarize this result in the following proposition.

Proposition 1. *For any vector of prices $\boldsymbol{P} = (p_k^i, p_{2m}^i)_{i\in\mathcal{N}}^{k\in\mathcal{S}}$, there exists at least one participation equilibrium $\boldsymbol{n}(\boldsymbol{P}) = (n_k^i, n_2^m)_{i\in\mathcal{N}}^{k\in\mathcal{S}}$. Moreover, if $\overline{q}\overline{\Phi} < 1/2$, then the PE is unique.*[15]

Proposition 1 shows that given any vector of prices, each consumer joins the platform that offers the highest utility, while each content provider chooses between joining a platform exclusively or multi-homing to all platforms, choosing the option that provides it the highest utility. Proposition 1 shows that uniqueness in the existence of a PE requires users from each side (as a whole) to be insensitive toward changes in participation decisions

[14]Let $\boldsymbol{u} = (u^1, \cdots, u^n, u^m)$ and $\boldsymbol{u} \in \mathbb{R}^{n+1}$. Note that as $u^m \to -\infty$, then $Q_2^m(\boldsymbol{u}) \to 0$ and $Q_2^i(\boldsymbol{u}) \to \mathbb{P}\left(u^i + \varepsilon_2^i \geq \max_{j\neq i,\, j\in\mathcal{N}}\left(u^j + \varepsilon_2^j\right)\right)$. That is, as $u^m \to -\infty$ our model converges to the exclusive-only game studied by Tan and Zhou (2020).

[15]The proof of Proposition 1 is very similar to the proof of Proposition 1 in Tan and Zhou (2020). We need to extend the proof in Tan and Zhou (2020) to account for equation (11) and the extra dimension in the utility vector $\boldsymbol{u} \in \mathbb{R}^{n+1}$—the utility of content providers choosing multi-homing. We omit this proof but is available upon request to the authors.

of users from the other side. This condition is satisfied either when $\overline{q}$ is low, which requires user participation to be insensitive toward changes in utilities offered by each platform; or when $\overline{\Phi}$ is low, which requires network benefits offered by platforms to be insensitive toward changes in participation of other-side users. The two conditions are complementary: if many users change participation decisions toward changes in utility offered by platforms, then condition $\overline{q}\overline{\Phi} < 1/2$ in Proposition 1 can be satisfied by having network benefit functions be insensitive toward changes in other-side users, and vice versa.[16]

3.2 Stage 1

In this section, we solve platforms' problem (7), taking into account users' responses. We assume the existence of a symmetric equilibrium (SE) where all platforms have the same mass of users and charge the same prices. Let $\boldsymbol{p}^* = (p_1^*, p_2^*, p_{2m}^*)$ be the equilibrium vector price, and let $\boldsymbol{n}^* = \left(n_1^* = \frac{1}{n}, n_2^* = \frac{1}{n}(1-x), n_2^{m*} = x\right)$ be the vector of equilibrium market shares. Note that in this SE, consumers split equally among all platforms, while a portion x of content providers multi-home and the remaining portion $1-x$ splits equally among all platforms. We derive the profit function of a platform that deviates from the proposed SE and show that this deviation is not profitable.

A platform can deviate either by changing its prices or by changing its market shares. We suppose that platform 1 deviates from the proposed SE by choosing a vector of market shares $\boldsymbol{n}^1 = (n_1, n_2, n_2^m)$. The other $n-1$ platforms adhere to the SE by splitting the remaining market: on side 1, each platform equally splits the remaining market share and each receives $\frac{1-n_1}{n-1}$; on side 2, the deviating platform chooses n_2 and n_2^m, and the other platforms split the remainder as single-homing content providers equally, each receiving $\frac{1-n_2-n_2^m}{n-1}$.[17] Given the vector of market shares, $\boldsymbol{n}^1$, we need to find a vector of prices, $\boldsymbol{p}^1$, such that $\boldsymbol{n}^1$ is well defined. That is, we need to find a well-defined mapping $\boldsymbol{n}^1 \mapsto \boldsymbol{p}^1(\boldsymbol{n}^1) = (p_1(\boldsymbol{n}^1), p_2(\boldsymbol{n}^1), p_{2m}(\boldsymbol{n}^1))$. If there exists a well-defined mapping $\boldsymbol{n}^1 \mapsto \boldsymbol{p}^1(\boldsymbol{n}^1)$, then it follows that $\boldsymbol{n}^1$ is a PE of $\boldsymbol{p}^1 = \boldsymbol{p}^1(\boldsymbol{n}^1)$ if and only if $(\boldsymbol{n}^1, \boldsymbol{p}^1)$ solves the following sets of equations:[18]

[16] The size of $\overline{q}\overline{\Phi}$ is capturing the degree in which system (12) is a contracting mapping.

[17] Note that sum of individual market shares equals that of the full market

$$n_1 + (n-1)\cdot\frac{1-n_1}{n-1} = 1 \text{ and } n_2 + (n-1)\cdot\frac{1-n_2-n_2^m}{n-1} + n_2^m = 1.$$

Also, platform i can "choose" n_2^m for everyone by unilaterally changing the total payment required to multi-homing $\sum_i p_{2m}^i$.

[18] Note that from Proposition 1, we know that there is a well-defined mapping $\boldsymbol{p}^1 \mapsto \boldsymbol{n}^1(\boldsymbol{p}^1)$. Below, we show sufficient conditions under which there is a well-defined mapping $\boldsymbol{n}^1 \mapsto \boldsymbol{p}^1(\boldsymbol{n}^1)$. Also note that in the exclusive-only game, this mapping is explicitly defined (see Tan and Zhou, 2020).

$$\begin{cases} n_1 & = Q_1^1\left(u_1^1, u_1^*, \ldots, u_1^*, 0\right) \\ n_2 & = Q_2^1\left(u_2^1, u_2^*, \ldots, u_2^*, u_2^m\right) \\ n_2^m & = Q_2^m\left(u_2^1, u_2^*, \ldots, u_2^*, u_2^m\right), \end{cases} \quad (13) \quad \text{and} \quad \begin{cases} \frac{1-n_1}{n-1} & = Q_1^j\left(u_1^1, u_1^*, \ldots, u_1^*, 0\right) \\ \frac{1-n_2-n_2^m}{n-1} & = Q_2^j\left(u_2^1, u_2^*, \ldots, u_2^*, u_2^m\right) \\ \text{for each} & j \in \{2, \cdots, n\}. \end{cases} \tag{14}$$

Equations (13) and (14) determine the market share of the deviating platform and non-deviating platforms, respectively. Note that u_1^1 is the deterministic utility component of a consumer who joins platform 1, and u_2^1 the deterministic utility component of a content provider that joins platform 1, and these are defined, respectively, as

$$u_1^1 \equiv v_1 + \phi_1\left(n_2, n_2^m\right) - p_1^1 \quad \text{and} \quad u_2^1 \equiv v_2 + \phi_2\left(n_1\right) - p_2^1.$$

u_1^* and u_2^* are the deterministic utility components of consumers and content providers, respectively, that join one of the $n-1$ non-deviating platforms, and are defined as

$$u_1^* \equiv v_1 + \phi_1\left(\frac{1-n_2-n_2^m}{n-1}, n_2^m\right) - p_1^* \quad \text{and} \quad u_2^* \equiv v_2 + \phi_2\left(\frac{1-n_1}{n-1}\right) - p_2^*.$$

Finally, the quantity u_2^m is the deterministic utility component of content providers multi-homing to all platforms, and is defined as

$$u_2^m \equiv v_2 + v_2\left(n-1\right)\delta + \phi_m\left(n_1, \frac{1-n_1}{n-1} \cdot \mathbf{1}_{n-1}\right) - p_{2m}^1 - \left(n-1\right)p_{2m}^*.$$

The profits of platform 1 can be rewritten as a function of $\boldsymbol{n}^1 = (n_1^1, n_2^1, n_2^m)$,

$$\pi^1\left(\boldsymbol{n}^1; \boldsymbol{p}^*\right) = n_1 \cdot p_1(\boldsymbol{n}^1) + n_2 \cdot p_2(\boldsymbol{n}^1) + n_2^m \cdot p_{2m}(\boldsymbol{n}^1).$$ [19]

The deviating platform chooses the vector of market shares $\boldsymbol{n}^1 = (n_1, n_2, n_2^m)$ that maximizes its benefits and a vector of prices $\boldsymbol{p}^1$ that is consistent with equations (13) and (14), taking all other platforms' prices as given.

Let us show that the mapping $\boldsymbol{n}^1 \mapsto \boldsymbol{p}^1(\boldsymbol{n}^1)$ is well-defined. Let $\beta \equiv u_2^m - u_2$ be the difference between the deterministic utility components of multi-homing and single-homing content providers, which is equal to

$$\beta = v_2\left(n-1\right)\delta + \phi_m\left(n_1, \frac{1-n_1}{n-1} \cdot \mathbf{1}_{n-1}\right) - \phi_2\left(n_1\right) + p_2^1 - p_{2m}^1 - \left(n-1\right)p_{2m}^*. \tag{15}$$

[19] Prices $p_1(\boldsymbol{n}^1)$, $p_2(\boldsymbol{n}^1)$, and $p_{2m}(\boldsymbol{n}^1)$ are in fact functions of $\boldsymbol{p}^*$ as well, $p_1(\boldsymbol{n}^1; p^*)$, $p_2(\boldsymbol{n}^1; p^*)$, and $p_{2m}(\boldsymbol{n}^1; p^*)$, respectively. We omitted it for presentation clarity.

Note that if $\beta \geq 0$ in equilibrium, then content providers would obtain larger deterministic utility by multi-homing than by single-homing. If in addition, the support of the distribution function of the idiosyncratic preferences of content providers is positive (e.g., if preferences are distributed following an exponential or uniform distribution over the interval $[0,1]$), then multi-homing strictly dominates single-homing—the equilibrium corresponds to a corner solution in which no content provider single-homes, $n_2^* = 0$, and all of them multi-home, $n_2^{m*} = 1$. Thus, if the support of the distribution function of the idiosyncratic preferences of content providers is positive, in an interior equilibrium, $\beta < 0$.[20] We show that there exist functions $M_2 : \mathbb{R} \longrightarrow \mathbb{R}$ and $M_{2m} : \mathbb{R} \longrightarrow \mathbb{R}$ such that the mapping $\boldsymbol{n}^1 \mapsto \boldsymbol{p}^1(\boldsymbol{n}^1)$ exists.[21] Moreover, (13) and (14) uniquely determine the mapping $\boldsymbol{n}^1 \mapsto \boldsymbol{p}^1(\boldsymbol{n}^1)$ if and only if (i) there is a unique $\beta^* \in \mathbb{R}$ such that

$$g(\beta^*) = 0, \tag{16}$$

where g is defined as

$$g(\beta) \equiv v_2(n-1)\delta + \phi_m\left(\frac{1}{n} \cdot \mathbf{1}_n\right) - \phi_2\left(\frac{1}{n}\right) + M_2(\beta) - nM_{2m}(\beta) - \beta;$$

and (ii) β^* is such that the Jacobian associated with equations (13) at $(\boldsymbol{n}^*, \boldsymbol{p}^*)$ is non-zero. Using the mapping $\boldsymbol{n}^1 \mapsto \boldsymbol{p}^1(\boldsymbol{n}^1)$, and from the first-order conditions of π^1, it follows that $\pi^1(\boldsymbol{n}^1; \boldsymbol{p}^*)$ has a stationary point $\boldsymbol{n}^* = \left(n_1^* = \frac{1}{n}, n_2^* = \frac{1}{n}(1-x), n_2^{m*} = x\right)$, where $x = x(\beta^*)$, and the price $\boldsymbol{p}^*$ is:[22]

$$\begin{aligned} p_1^* &= \underbrace{\frac{1 - H_1(0)}{h_1(0)}}_{\equiv M_1(0)} - \frac{1}{n-1}\phi_2'\left(\frac{1}{n}\right), \\ p_2^* &= M_2(\beta^*) - \frac{1}{n-1}\frac{\partial \phi_1}{\partial n_2^i}\left(\frac{1}{n}(1-x), x\right), \text{ and} \\ p_{2m}^* &= M_{2m}(\beta^*) - \frac{1}{n(n-1)}\frac{\partial \phi_1}{\partial n_2^i}\left(\frac{1}{n}(1-x), x\right). \end{aligned} \tag{17}$$

The terms M_1, M_2, and M_{2m} represent the market power held by platforms in each market. The negative terms in (17) associated with either $\partial\phi_1/\partial n_2^i$ or ϕ_2' are the subsidies received by the users. Note that these terms are determined by the number of platforms in the market, n, and the size of the derivatives of the network benefit functions at the

[20] In Section 4, we assume that users' idiosyncratic preferences follow either an exponential or uniform distribution with a positive support, and show that there is a unique interior equilibrium in which $\beta^* < 0$.

[21] These functions are uniquely determined by the distribution of content providers' idiosyncratic preferences. See the proof of Theorem 1 for a definition of $M_2(\cdot)$ and $M_{2m}(\cdot)$.

[22] A precise definition of $x(\cdot)$, $H_1(\cdot)$, and $h_1(\cdot)$ can be found in the proof of Theorem 1.

equilibrium values. To ensure that any deviation made by platform 1 is non-profitable, we need to guarantee that $\boldsymbol{n}^*$ is a global maximizer of π^1.

Assumption 1. *Every stationary point of* $\pi^1(\boldsymbol{n}^1; p^*)$ *on* $[0,1]^3$ *is a global maximum point, where* $\boldsymbol{p}^*$ *is given by (17).*

Assumption 1 imposes sufficient conditions on the profit function of the deviating platform to have a unique global maximizer. In Lemma 1, we provide simpler conditions—on the distributions of the idiosyncratic preferences—which imply Assumption 1. These conditions can be checked ex-post once we impose a functional form on the idiosyncratic preferences.

Theorem 1. *Under full market coverage and Assumption 1, suppose there exists a unique solution* β^* *of the equation (16) and that* $J(\beta^*) \neq 0$*, where* J *is the Jacobian associated with equations (13). Then, there exists a subgame perfect Nash equilibrium where all platforms charge prices* $\boldsymbol{p}^* = (p_1^*, p_2^*, p_{2m}^*)$ *given by (17), and the equilibrium market shares are given by* $\boldsymbol{n}^* = \left(\frac{1}{n}, \frac{1}{n}\left[1 - x\left(\beta^*\right)\right], x\left(\beta^*\right)\right)$.

Theorem 1 shows that equilibrium prices follow an additively separable structure similar to the exclusive-only case discussed by Tan and Zhou (2020). It shows that both the exclusive and non-exclusive prices can be decomposed into a market power term (M) and a network subsidy term (ϕ_k'). Finally, Theorem 1 shows that the solution of the subgame perfect Nash equilibrium can be reduced to show that there is a unique β that satisfies $g(\beta) = 0$ and that the Jacobian associated with equations (13) at β^* is non-zero.

Proposition 2. *Under full market coverage and Assumption 1, suppose there exists a unique solution* β^* *of the equation (16) and that* $J(\beta^*) \neq 0$*, where* J *is the Jacobian associated with equations (13). Suppose the support of the distribution function of the idiosyncratic preferences of content providers is positive. If* $(\boldsymbol{n}^*, \boldsymbol{p}^*)$ *is an interior equilibrium (i.e.,* $n_1^* > 0$*,* $n_2^* > 0$*, and* $n_2^{m*} > 0$*), then* $\beta^* < 0$.

From Proposition 2, it follows that if $\beta^* \geq 0$ and the support of the distribution function of the idiosyncratic preferences of content providers is positive, then multi-homing strictly dominates single-homing and the equilibrium corresponds to a corner solution in which no content provider single-homes, $n_2^* = 0$, and all of them multi-home, $n_2^{m*} = 1$. On the other side, as $\beta^* \to -\infty$, then single-homing strictly dominates multi-homing and the equilibrium corresponds to a corner solution in which no content provider multi-homes, $n_2^{m*} = 0$, and all of them single-home, $\sum_{i=1}^n n_2^* = 1$.

In an interior equilibrium there exists a set of cutoffs, $\{u^i\}_{i=1}^{n+1}$, such that content providers for which $\sum_{j \neq i} \varepsilon_2^j \geq u^i - u^{n+1}$ for all $i = 1, \ldots, n$ choose to multi-home to all platforms. That is, content providers with high idiosyncratic preferences for all platforms will multi-home. Similarly, content providers for which there exists $i \in \{1, \ldots, n\}$ such that $\sum_{j \neq i} \varepsilon_2^j < u^i - u^{n+1}$ and $\varepsilon_2^i - \varepsilon_2^j \geq u^j - u^i$ for all $j \neq i$ will single-home to platform i.

3.3 Discussion

The introduction of non-exclusive contracts in the content providers' side leads to a new margin: the exclusive–non-exclusive margin that platforms base their pricing decisions on. Replacing an exclusive–exclusive margin with an exclusive–non-exclusive margin softens platforms' competition for content providers. Consider a market with two platforms i and j. If a mass λ of exclusive content providers on platform i change their participation decision to multi-home on both platforms, then platform j gains an advantage of λ over platform i. If instead the content providers become exclusive content providers on another platform, then platform j experiences a net gain of 2λ in content provider advantage over platform i. Hence, the introduction of non-exclusive contracts leads to a higher market power component, M_2, in the content provider single-homing price (see, e.g., Corollary 2).

The marginal network benefit for consumers from adding exclusive content providers to platform i is denoted as $\frac{\partial \phi_1}{\partial n_2^i}$. The exclusive and non-exclusive content provider subsidies depend on how much additional exclusive content providers are worth to consumers, but not on the value of additional non-exclusive content providers. If platforms were interested in increasing their total available content—without necessarily increasing their advantage over other platforms—then we would expect the term $\frac{\partial \phi_1}{\partial n_2^m}$ to be part of the subsidy terms. The presence of the marginal exclusive content providers' valuation of consumers and the lack of the marginal non-exclusive content providers' valuation indicates that the subsidy is a tool used to attract content providers that can provide an advantage over other platforms.

Let us recall from Section 2 that the network benefit functions of side k depend only on the market shares of side l for $k, l \in \{1, 2\}$ and $k \neq l$ (i.e., for side 1 we have $\phi_1 = \phi_1(n_2^i, n_2^m)$; for side 2, we have $\phi_2 = \phi_2(n_1^i)$ and $\phi_m = \phi_m(n_1^1, \ldots, n_1^n)$). In the special case with exclusive-only content providers (i.e., $\beta \to -\infty$), the equilibrium prices have subsidy terms that are functions of only the other-side marginal network benefits, $\frac{\partial \phi_k}{\partial n_l^i}$ for $k, l \in \{1, 2\}$, $i \in \mathcal{N}$, and $k \neq l$.[23] When we introduce non-exclusive contracts, content providers' prices now depend on the own-side network benefit functions ϕ_2, ϕ_m through β^* (see, e.g., equation (15)) in addition to cross-side network benefits ϕ_1.[24] Intuitively, consumers prefer the platform with the highest number of content providers available, and since

[23]A similar structure is found in prices in Armstrong (2006).

[24]Tan and Zhou (2020) explained the term $\frac{\partial \phi_k}{\partial n_l^i}$ as marginal profit of additional users, which comple-

multi-homing content providers are shared by all platforms, consumers' choices between platforms ultimately depend on the number of exclusive content providers available on each platform. The share of exclusive versus non-exclusive content providers is determined by β^*, which is a function of the network benefit functions ϕ_2 and ϕ_m. Thus, platforms' ability to attract consumers depends on the share of exclusive content providers available on the platform, explaining why content providers' prices depend on the own-side network benefit function ϕ_2, ϕ_m.

If the network benefit function ϕ_1 is linear, the $1/(n-1)$ coefficient in the subsidy terms implies that as the number of platforms increases, both single-homing and multi-homing content provider subsidies decrease. Tan and Zhou (2020) explain the coefficient as the marginal advantage gained by increasing 1 unit in users. When there are n platforms, an increase in 1 unit of users is equivalent to removing $1/(n-1)$ user from each competitor, resulting in $2/(n-1)$ user net advantage over each competitor. The coefficient represents the increased difficulty in gaining net advantage as the number of competitors increases. While this is counterintuitive to one-sided markets, Wright (2004) pointed out that two-sided markets often deviate from the intuitions we have with one-sided markets.[25]

Finally, in an interior equilibrium, content providers receive the same total subsidy regardless of their choice of exclusivity. The exclusive content providers receive the full subsidy from the single platform they join, $\frac{1}{n-1}\frac{\partial \phi_1}{\partial n_2^i}$, while non-exclusive content providers receive $1/n$ of this term, $\frac{1}{n-1}\frac{\partial \phi_1}{\partial n_2^i}$, from each of the n platforms they join. This result, combined with the fact that β^* is independent of ϕ_1, implies that changes in the consumer network benefit function ϕ_1 have no impact on the share of exclusive versus non-exclusive content providers. Our model predicts that as consumers increase their marginal valuation for additional content (due to changes in tastes or improvement in content recommendation systems [e.g., *Netflix* learns your preferences and recommend videos you are more likely to watch]), it will impact the payment that content providers receive, but the difference in the total payments that exclusive and non-exclusive content providers receive will stay the same, leaving the decision to single-home or multi-home unchanged.[26] To summarize, the content provider's homing decision is neutral to changes in consumer network benefit function.

ments our interpretation of marginal network benefit for $\frac{\partial \phi_1}{\partial n_2^i}$. Each time a platform steals a unit of users from competing platforms, the loss is shared equally among all other platforms. When the number of competitors goes up, it lowers the payoff from undercutting prices, resulting in higher prices for users.

[25] Wen and Zhu (2019) showed empirically that platform entry resulted in higher prices charged to content providers.

[26] Here we are referring to the difference $p_2^* - np_{2m}^*$ and not the exclusive premium.

4 Examples: Exponential and Uniform Distributions

In this section, we study two examples where equation (16) has a unique solution and satisfy the condition that guarantees the existence of a subgame perfect Nash equilibrium in Theorem 1, and we compare the equilibrium prices and market shares with the exclusive-only game. We will focus on the case of three platforms, $n = 3$, and linear network benefit functions—$\phi_1(n_2^i, n_2^m) = a_1(n_2^i + n_2^m)$, $\phi_2(n_1^i) = a_2 n_1^i$, and $\phi_m\left(n_1^1, n_1^2, n_1^3\right) = a_2\left(n_1^1 + n_1^2 + n_1^3\right)$, where a_1, a_2 are positive constants.[27]

4.1 Exponential distribution

In this subsection, we assume that for each $i \in \{1, 2, 3\}$ and $k \in \{1, 2\}$, ε_k^i follows an exponential distribution with parameter $r_k > 0$; that is, $\varepsilon_k^i \sim \text{Exp}(r_k)$. Note that $\mathbb{E}[\varepsilon_k^i] = \frac{1}{r_k}$ is the expected value of the random utility component of side k users joining platform i.[28] For each side of the market, $k \in \{1, 2\}$, the idiosyncratic preferences $\{\varepsilon_k^1, \varepsilon_k^2, \varepsilon_k^3\}$ are i.i.d. We show that equation (16) has a unique solution and provide the equilibrium prices and market shares of the game.

We introduce the following assumption to ensure that the second-order conditions are satisfied:

Assumption 2. *(i)* $\frac{1}{r_2} > \frac{1}{3}a_2 + \delta v_2$*;*
(ii) $\frac{4}{3r_1r_2} > (a_1 + a_2)^2$.

Let $v_0 \equiv \frac{2}{3}a_2 + 2\delta v_2$. From (15), v_0 is the difference between the deterministic utility components of multi-homing and single-homing content providers when the prices charged are equal to zero (i.e., $u_2^m - u_2$ at $p_2^i = p_{2m}^i = 0$ for $i \in \{1, 2, 3\}$). Similarly, $\frac{2}{r_2}$ is the expected value of the difference between the random utility components of multi-homing and single-homing content providers. Assumption 2(i) says that the expected value of the difference between the random utility components of multi-homing and single-homing content providers is greater than the difference between the deterministic utility components of multi-homing and single-homing content providers when the prices charged are equal to zero. Assumption 2(ii) says that the product of the expected values of the random utility component of consumers and content providers $(\frac{1}{3r_1r_2})$ is larger than the square of the sum of the subsidy terms for consumers and content providers (i.e., $\frac{a_1}{2}$ and $\frac{a_2}{2}$). This assumption guarantees that when content providers are charged non-zero

[27] In Appendixes C and D, we provide the details of the model with two platforms when the idiosyncratic preferences follow an exponential and uniform distribution, respectively.

[28] If r_k is small, then the expected user idiosyncratic preferences for platforms are stronger and the random utility component becomes more relevant than the deterministic utility component. Thus, if r_2 is small, content providers will be less likely to multi-home, as we show in Corollary 1.

prices, the network content providers effect does not always dominate the idiosyncratic preferences, guaranteeing that the second-order conditions are satisfied.[29] We suppose this assumption is satisfied for the rest of this subsection.

Proposition 3. *There exists a unique $\beta^* < 0$ such that equation (16) holds, implicitly defined by*

$$g(\beta) = v_0 - \frac{1}{2r_2}\frac{4 - 3e^{\frac{r_2\beta}{2}}}{1 - e^{\frac{r_2\beta}{2}}} - \beta = 0.$$

Moreover, a unique symmetric subgame perfect Nash equilibrium exists in which, in stage 1, all platforms charge prices

$$\begin{aligned} p_1^* &= \underbrace{\frac{1}{r_1}}_{M_1(0)} - \frac{a_2}{2}, \\ p_2^* &= \underbrace{\frac{1}{r_2\left(1 - y^4\right)}}_{M_2(\beta^*)} - \frac{a_1}{2}, \text{ and} \\ p_{2m}^* &= \underbrace{\frac{-3y^4 + y^3 + y^2 + y + 6}{6r_2\left(1 - y^4\right)}}_{M_{2m}(\beta^*)} - \frac{a_1}{6}, \end{aligned} \tag{18}$$

where $y \equiv e^{\frac{\beta^* r_2}{2}} \in (0,1)$. In stage 2, the market shares are $n_1^* = \frac{1}{3}$, $n_2^* = \frac{1}{3}\left(1 - y^3\left(4 - 3y\right)\right)$, and $n_2^{m*} = y^3\left(4 - 3y\right)$.

Note that, from Proposition 3, the market power term M_{2m} is larger than M_2 for any value of $y \in (0,1)$, which implies that platforms exert higher market power on multi-homing content providers. The subsidy term in p_2^*, $a_1/2$, is three times that of p_{2m}^*, $a_1/6$, so content providers receive the same subsidy regardless of their decision to single-home or multi-home, as noted in the previous section.

From Proposition 3, it follows that $\beta^* = u_2^{m*} - u_2^* < 0$; then, single-homing content providers get a strictly higher deterministic utility than those multi-homing.[30] Moreover, from Proposition 3 and equation (15),

[29] Note that if the idiosyncratic preferences follow an exponential distribution, from Proposition 2, it follows that $\beta^* < 0$ is a necessary condition for $(\boldsymbol{n}^*, \boldsymbol{p}^*)$ to be an interior equilibrium. Assumption 2 guarantees that in fact $\beta^* < 0$, and that $(\boldsymbol{n}^*, \boldsymbol{p}^*)$ is an interior equilibrium.

[30] Suppose that $\beta \geq 0$ in equilibrium; then, content providers obtain larger deterministic utility by multi-homing than by single-homing. Given that the idiosyncratic preferences of content providers are positive, then multi-homing strictly dominates single-homing. It follows that no one single-homes, $n_2^* = 0$, and everyone multi-homes, $n_2^{m*} = 1$. This is similar to the strong differentiation on the one-side model in Armstrong and Wright (2007). However, we show in the proof of Proposition 3 that there is no equilibrium for which $\beta \geq 0$, since this requires setting some prices equal to infinite.

$$3p_{2m}^* - p_2^* > v_0. \tag{19}$$

Inequality (19) is a necessary condition for an interior equilibrium. It shows that the difference between three times the multi-homing prices and single-homing prices, $3p_{2m}^* - p_2^*$, has to be greater than $(u_2^m - u_2)|_{p_{2m}=p_2=0} = v_0$—the difference between the deterministic utility components of multi-homing and single-homing content providers when the prices charged are equal to zero—so that content providers do not always choose to multi-home.

Note that in equilibrium, $\beta^* < 0$, which is different from the incentive compatibility constraint. Remember that β^* is equal to the difference between the deterministic utility components for multi-homing and single-homing content providers, whereas the incentive compatibility constraint compares content providers' prices for multi-homing and single-homing. That is, $p_2^* - 3p_{2m}^* < 0$ does not imply $p_2^* - p_{2m}^* < 0$.

The following corollary shows how the equilibrium market shares change with the content provider differentiation parameter r_2.[31]

Corollary 1. *In an exclusive–non-exclusive equilibrium:*

(i) $\frac{\partial \beta^*}{\partial r_2} > 0$;

(ii) $\frac{\partial n_2^{m*}}{\partial r_2} > 0$;

Part (i) shows that β^* is decreasing in $\frac{1}{r_2}$; that is, content providers' utility to multi-home decreases as platform differentiation to content provider increases. In other words, as the expected value of the random utility component of content providers increases (higher $\frac{1}{r_2}$), content providers have stronger preferences for platforms, which makes single-homing more attractive. Part (ii) follows as a consequence of (i): As $\frac{1}{r_2}$ increases, content providers are less likely to multi-home.

Comparison to the Exclusive-Only Game. Using the exclusive-only model as a benchmark, we compare the changes in prices and social surpluses upon the introduction of non-exclusive prices. The following corollary shows that in the exponential setting for three platforms, the prices and market shares of the exclusive-only model can be obtained as a limit case of the exclusive–non-exclusive model.

Corollary 2. *As* $\beta^* \to -\infty$, $p_1^* \to p_{1,E}^* \equiv \frac{1}{r_1} - \frac{a_2}{2}$, $p_2^* \to p_{2,E}^* \equiv \frac{1}{r_2} - \frac{a_1}{2}$, $n_2^* \to n_{2,E}^* \equiv \frac{1}{3}$ *and* $n_2^{m*} \to 0$. *Moreover,* p_2^* *monotonically decreases to* $p_{2,E}$ *as* $\beta^* \to -\infty$.

[31] Note that content provider prices and market shares are independent of r_1. Moreover, as r_1 increases, equilibrium consumer price, p_1^*, decreases. Similarly, as r_2 increases, equilibrium content provider prices (exclusive and non-exclusive) decrease.

Corollary 2 shows that prices and market shares for the exclusive–non-exclusive game converge to the prices and markets shares of the exclusive-only game (see, for example, Tan and Zhou, 2020).[32] The introduction of non-exclusive contracts softens the competition for content providers; that is, the market power component of exclusive content provider price increases from $1/r_2$ to $1/r_2(1-y^4)$.[33] The subsidy term of the content provider price does not change as a result of changing exclusive content availability because of the linearity of the network benefit function.[34] Overall, the content provider single-homing price is higher after introduction of non-exclusive contracts.

Notice that consumer price is the same in both games. Consumer price depends on the market power term $M_1 = 1/r_1$ that platforms exert on consumers minus the subsidies consumers receive for attracting content providers $a_2/2$. The market power M_1 is driven by the single-sided market competition for consumers between platforms. Note that the competition for consumers does not change when exclusive–non-exclusive contracts are introduced; that is, the market power term remains as $1/r_1$. The subsidy consumers receive, $a_2/2$, depends on how much the marginal consumer is worth to the single-homing content provider, $\frac{\partial \phi_2}{\partial n_1}$, which stays the same in both games.

Why doesn't the non-exclusive content provider network benefit affect the consumer subsidy term? To increase the content provider advantage over their rivals, platforms offer subsidies to both exclusive and non-exclusive content providers. The difference in consumer network benefit offered by platforms i, j is given by $a_1(n_2^i + n_2^m) - a_1(n_2^j + n_2^m)$. Increasing n_2^m does not impact the difference. Only by increasing single-homing content providers, n_2^i, can platforms increase their advantage. Thus, only the single-homing content providers' network benefit matters.

We now compare changes in social surpluses upon the introduction of non-exclusive contracts in the following corollary.

Corollary 3. *The introduction of non-exclusive contracts:*

(a) *increases platform profits and consumer surplus, increases the surplus of content providers that multi-home, but decreases the surplus of those that single-home;*

(b) *increases (decreases) the sum of consumers' and content providers' surpluses when $v_0 \to 0$ ($v_0 \to \infty$);*

[32] This result follows from our claim in footnote 14. As $\beta^* \to -\infty$, our model converges to the exclusive-only game studied by Tan and Zhou (2020); i.e., $n_2^{m*} = 0$. When $\beta^* \geq 0$, we get a corner equilibrium in which all content providers multi-home, $n_2^{m*} = 1$.

[33] In the exclusive-only game, the market power component is equal to $1/r_2$.

[34] Note that, from (17), the subsidy term of the content provider price may change as a result of changing exclusive content availability if the network benefits are nonlinear, since $\frac{\partial \phi_1}{\partial n_2^i}$ depends on $x(\beta^*)$.

(c) *increases the overall welfare when either* $v_0 \to 0$ *or* $v_0 \to \infty$.[35]

Corollary 3 shows that platforms and consumers gain from the introduction of non-exclusive prices. Consumers pay the same price as in the exclusive-only game, but gain access to the new content provided by multi-homers. Platforms' profits increase by two means: the higher price being charged to exclusive content providers ($p_2^* > p_{2,E}^*$, see Corollary 2); and the gains from charging more to non-exclusive content providers than to exclusive content providers ($3p_{2m}^* > p_2^*$, see inequality (19)).

After the introduction of non-exclusive contracts, content providers are ex-post divided into single-homers and multi-homers. Content providers remaining as single-homers now pay more, while maintaining access to the same mass of consumers, resulting in a loss. Content providers that become multi-homers pay a higher price than single-homers but gain access to more consumers and receive multiple realizations of positive idiosyncratic preferences. Multi-homing content providers are better off since the gain from these positive realizations of idiosyncratic preferences outweighs the exclusive premium payment ($p_{2m}^* - p_2^* > 0$).

Part (b) shows that the overall change in consumer and content provider surplus is ambiguous. While consumer surplus always increases upon the introduction of non-exclusive prices, exclusive content provider surplus can decrease more than the consumer surplus can increase. For instance, if the difference between the deterministic utility components of multi-homing and single-homing content providers when the prices charged are equal to zero is small (i.e., v_0 is close to zero), then the increase in consumer and non-exclusive content provider surpluses is high enough to outweigh the loss in exclusive content provider surplus. The opposite case is true: if v_0 is large (i.e., $v_0 \to \infty$), so that there are too many multi-homers in the market (y close to 1), then the loss in exclusive content provider surplus is high enough to outweigh the gains in consumer and non-exclusive content provider surpluses.

In part (c), we show that the overall welfare increases when either $v_0 \to 0$ or $v_0 \to \infty$. It follows that platforms', consumers', and multi-homing content providers' gains, after the introduction of non-exclusive contracts, are enough to outweigh the exclusive content provider losses when either $v_0 \to 0$ or $v_0 \to \infty$. Thus, from Corollary 3, it follows that the introduction of non-exclusive contracts is a welfare enhancing tool.

Note that for each platform, the non-exclusive price becomes a surplus extraction

[35] Unfortunately, we do not have a formal proof of this result for $v_0 \in (0, \infty)$. Our simulations discussed in the next section suggest that the overall welfare increases for any value of $v_0 > 0$ (see Figure B2 in the Appendix B). Corollary C.1 shows that in the case of two platforms, the introduction of non-exclusive contracts is a welfare enhancing tool, since in that case, the platform profits, consumer surplus, and surplus of multi-homing content providers increase, and the surplus of single-homing content providers remains constant.

tool. It allows platforms to have more instruments to extract content provider surplus.

4.2 Uniform distribution

In this subsection, we assume uniform idiosyncratic preferences. Most of the results provided for the model with exponential idiosyncratic preferences in the previous subsection hold, so we limit our discussion to the existence of the equilibrium. In Appendix D, we study additional properties of the equilibrium prices and market shares.

We assume that for each $i \in \{1,2,3\}$ and $k \in \{1,2\}$, ε_k^i follows an uniform distribution with parameter $t_k > 0$; that is, $\varepsilon_k^i \sim \mathrm{U}[0, t_k]$. Note that $\mathbb{E}[\varepsilon_k^i] = \frac{t_k}{2}$ is the expected value of the random utility component of side k users joining platform i. For each side of the market, $k \in \{1,2\}$, the idiosyncratic preferences $\{\varepsilon_k^1, \varepsilon_k^2, \varepsilon_k^3\}$ are i.i.d. The following proposition characterizes the symmetric equilibrium.

Proposition 4. *Assume that $t_2 > \frac{1}{3}a_2 + \delta v_2$ and $\frac{32}{27}t_1 t_2 > (a_1 + a_2)^2$. There exists a unique $\beta^* < 0$ such that equation (16) holds, implicitly defined by*

$$g(\beta) = \frac{2\left(\beta t_2\left(4a_2 - 15\beta + 12\delta v_2\right) + 3\beta^2\left(a_2 - 3\beta + 3\delta v_2\right) + 6t_2^3\right)}{3\beta\left(3\beta + 4t_2\right)} = 0.$$

Moreover, a unique symmetric subgame perfect Nash equilibrium exists in which, in stage 1, all platforms charge prices

$$\begin{aligned} p_1^* &= \underbrace{\frac{t_1}{3}}_{\equiv M_1(0)} - \frac{a_2}{2}, \\ p_2^* &= \underbrace{-\frac{t_2^2}{2\beta^*}}_{\equiv M_2(\beta^*)} - \frac{1}{2}a_1, \text{ and} \\ p_{2m}^* &= \underbrace{\frac{2\beta^{*3} - 4t_2^3 - \beta^* t_2^2 + 4\beta^{*2} t_2}{6\beta^{*2} + 8\beta^* t_2}}_{\equiv M_{2m}(\beta^*)} - \frac{1}{6}a_1. \end{aligned} \tag{20}$$

In stage 2, the market shares are $n_1^* = \frac{1}{3}$, $n_2^* = \frac{1}{3}(1-x)$, and $n_2^{m*} = x$, where $x = 1 - \frac{3\beta^{*2}(\beta^* + 2t_2)}{4t_2^3}$.

Note that, from Proposition 4, it follows that

$$M_{2m}(\beta^*) - M_2(\beta^*) = \frac{(\beta^* + t_2)^2}{3\beta^* + 4t_2}.$$

In the proof of Proposition 4, we show that $\beta^* \in (-t_2, 0)$. It follows that the market power term M_{2m} is larger than M_2, which implies that platforms exert higher market power on multi-homing content providers, as in the exponential distribution game. The subsidy term in p_2^*, $a_1/2$, is three times that of p_{2m}^*, $a_1/6$, so content providers receive the same subsidy regardless of whether they choose single-homing or multi-homing. In equilibrium, content providers obtain smaller deterministic utility through multi-homing rather than single-homing but are compensated with positive realizations of the idiosyncratic preferences $\{\varepsilon_2^1, \varepsilon_2^2, \varepsilon_2^3\}$. Moreover, inequality (19) holds for the uniform case. Then, it follows that content providers that multi-home are being charged more than those that single-home, with the difference in prices, $p_{3m}^* - p_2^*$, greater than the difference between the deterministic utility components of multi-homing and single-homing when the prices charged are equal to zero. Thus, the qualitative results of Proposition 3 hold for the game with uniform idiosyncratic preferences.[36]

In Corollary D.1 in Appendix D, we show that as the expected value of the random utility component of content providers increases (i.e., t_2 increases), β^* decreases (i.e., content providers have stronger preferences for platforms), increasing the attractiveness of the single-homing content providers for platforms. Thus, the proportion of multi-homing content providers decreases as t_2 increases.

Moreover, Corollary D.2 in Appendix D shows that the introduction of non-exclusive prices softens competition for content providers (i.e., $p_2^* > p_{2,E}^*$). As in the game with exponential idiosyncratic preferences, with the introduction of non-exclusive contracts, exclusive content providers' surplus decreases. The effect on surplus of content providers that switch to multi-homing depends on $3p_{2m}^* - p_{2,E}^*$, which is positive since $3p_{2m}^* > p_2^*$ and $p_2^* > p_{2,E}$. If $3p_{2m}^* - p_{2,E}^* \geq 2t_2 + v_0$—that is, if multi-homing content providers are being charged more than two times the expected value of the difference between the random utility components of multi-homing and single-homing content providers, $2t_2$, plus the difference between the deterministic utility components of multi-homing and single-homing content providers, v_0—then multi-homing content providers are worse off with the introduction of non-exclusive contracts. The opposite result holds true: If $3p_{2m}^* - p_{2,E}^* < 2t_2 + v_0$, then multi-homing content providers are better off with the introduction of non-exclusive contracts.

Corollary D.2 also shows that consumers benefit from the new content provided by multi-homers while paying the same price charged in the exclusive-only game. This implies that consumer surplus increases with the introduction of the non-exclusive price. Additionally, platforms' profits increase as a result of a softening in competition and gains from multi-homing content providers that are being charged more than single-homing ones. Finally, if the marginal network benefit a_1 is high, the gains in consumer surplus and platform profits are enough to outweigh the loss in content provider surplus, so that

[36] Note also that both distribution functions have positive supports.

the overall welfare increases. Thus, non-exclusive contracts increase welfare.[37]

Again, note that the qualitative results presented in Corollary 3 hold when the idiosyncratic preferences are uniformly distributed except for multi-homing content providers that, in certain cases, can be worse off with the introduction of non-exclusive contracts.

5 Entry

We now study how prices and platform profits change when the number of platforms goes from $n = 2$ to $n = 3$ in our exclusive–non-exclusive model. We assume that for each $i \in \{1, 2, 3\}$ and $k \in \{1, 2\}$, ε_k^i follows an exponential distribution with parameter $r_k > 0$. For $n \in \{2, 3\}$, in the exclusive–non-exclusive game with n platforms, $p_{1,n}^*$ denotes the price that consumers pay to join a platform. Note that

$$p_{1,3}^* - p_{1,2}^* = \frac{a_1}{2} > 0.$$

It follows that consumer price increases when the number of platforms goes from $n = 2$ to $n = 3$. This is because the subsidy that consumers receive in equilibrium decreases as the number of platforms increases, while the market power of platforms remains constant.[38]

For content providers, the price increases when the number of platforms increases from $n = 2$ to $n = 3$. There are two forces: the network and market power effects. For $n \in \{2, 3\}$, in the exclusive–non-exclusive game with n platforms, $p_{2,n}^*$ denotes the price that content providers pay to exclusively join a platform; $p_{2m,n}^*$ denotes the price content providers pay to each platform to non-exclusively join all platforms.[39] Note that

$$\begin{aligned} p_{2,3}^* - p_{2,2}^* &= \frac{a_1}{2} + \frac{y^4}{r_2 \left(1 - y^4\right)} > 0, \text{ and} \\ p_{2m,3}^* - p_{2m,2}^* &= \frac{a_1}{3} + \frac{3y^4 + y^3 + y^2 + y}{6r_2 \left(1 - y^4\right)} > 0. \end{aligned}$$

The introduction of non-exclusive contracts gives a new margin to platforms—the non-exclusive margin. Then, the market power that each platform exerts over content providers

[37] By a_1 high, we mean that there exists $\overline{a}_1 > 0$ such that for any $a_1 > \overline{a}_1$, the gains in consumer surplus outweigh the loss in content provider surplus (see the proof of Corollary D.2).

[38] In the exclusive–non-exclusive game with n platforms, consumer price is given by

$$p_{1,n}^* = \frac{1 - H_1(0)}{h1(0)} - \frac{1}{n-1} \phi_2' \left(\frac{1}{n}\right).$$

If we assume linear externalities, $p_{1,n}^* = \frac{1}{r_1} - \frac{1}{n-1} a_2$. Then, the consumer price increases as n increases since the subsidy that consumers receive, $\frac{1}{n-1} a_2$, is decreasing with respect to n.

[39] For $i \in \{1, 2, 2m\}$, $p_{i,3}^*$ is given by (18) and $p_{i,2}^*$ is given by (C.1) in Appendix C.

increases as n increases from two to three.

Note that when the number of platforms increases, each platform has a lower total market share of content providers, but prices increase. To simplify the comparison of profits, let $h : \mathbb{R}_+^2 \longrightarrow \mathbb{R}$ be such that

$$\pi_3^* > \pi_2^* \iff 2(a_1 + a_2) - \left(\frac{1}{r_1} + \frac{1}{r_2}\right) > h(v_0, r_2). \tag{21}$$

The function $h(v_0, r_2)$ summarizes the effect of introducing non-exclusive contracts on prices and market shares.[40]

Corollary 4. *Assume $\frac{1}{r_2} > \frac{1}{2}a_2 + \delta v_2$.[41] In the exclusive–non-exclusive game, if n goes from two to three platforms: (i) consumer and content provider prices increase, (ii) platforms profits increase if a_1 is high enough such that (21) holds, (iii) exclusive content provider surplus decreases, and (iv) non-exclusive content provider surplus decreases if $a_1 > 2\delta v_2$.*

Corollary 4 shows that the change in profits when the number of platforms increases from two to three (in the exclusive–non-exclusive game) can be broken down into the price effect and the market share effect. Prices are higher, but market shares are lower in the three-platform model. Note that if the network benefit a_1 is high enough such that (21) holds, the price effect dominates the market share effect. This implies that platforms make higher individual profit when there are three platforms in the market. In other words, when there are more platforms, each of them is more focused on extracting surplus from users (i.e., consumer and content provider prices increase) as opposed to increasing the market share (i.e., consumer and content provider market shares decrease).

Exclusive content providers are worse off due to the higher price and the smaller market share. Non-exclusive content providers pay more in the three-platform case but receive additional base utility. Note that multi-homers not only have to pay a higher non-exclusive price, but they also need to pay the price to an additional platform. Thus, non-exclusive content providers are worse off if the discounted base utility, δv_2, is smaller

[40] The function $h(v_0, r_2)$ depends on the parameters a_2, v_2, and δ through v_0. Note that h does not depend on $\{r_1, a_1\}$.

[41] Note that this assumption is stronger than Assumption 2, and it is required for the existence of an interior equilibrium in the exclusive–non-exclusive game with two platforms (see Appendix C). When the idiosyncratic preferences follow an exponential distribution with n platforms and linear profit functions from (15), it follows that Assumption 2 becomes $\frac{1}{r_2} > \frac{1}{n}a_2 + \delta v_2$. As n decreases, the right-hand side of this inequality increases, given that the marginal network benefit a_2 is divided among fewer platforms. The multi-homing choice becomes more attractive and a larger expected random utility component, $\frac{1}{r_2}$, is needed to guarantee the existence of an interior equilibrium.

than the marginal network benefit $\frac{a_1}{2}$. Recall that the subsidy term in the multi-homing content providers' price is $\frac{a_1}{2}$ when there are two platforms and $\frac{a_1}{6}$ when there are three platforms in the market. The latter case implies that the total subsidy for multi-homing content providers with $n = 2$ (i.e. a_1) is larger than the total subsidy when $n = 3$ (i.e. $\frac{a_1}{2}$), which explains the multi-homing content provider surplus loss when $a_1 > 2\delta v_2$.[42]

The results in Corollary 4 are similar for the exclusive-only game with exponential idiosyncratic preferences (see, for example, Tan and Zhou, 2020). That is, as the number of platforms increases, consumer and content provider prices increase, and platform profits increase as long as (21) holds with $h \equiv 0$—similar to the exclusive–non-exclusive game. However, consumers and content providers are worse off in the exclusive-only game, since in both sides of the market, prices increase and market shares decrease. As a result, the overall welfare (i.e., the sum of consumers' surplus, content providers' surplus, and platforms' profit) decreases when n goes from two to three platforms in the exclusive-only game.

Numerical Example. We now provide a numerical example to illustrate Corollary 4. We use equilibrium prices and market shares of the exclusive–non-exclusive game—equations (18) and (C.1)—when idiosyncratic preferences follow an exponential distribution for two and three platforms. We use the following parameters: $\delta = 0.9$, $v_2 = 1$, $r_1 = 0.4$, $a_2 = 2$, and $a_1 = 2$.

From Corollary 4, it follows that when then number of platforms increases in the exclusive–non-exclusive game, exclusive content provider surplus decreases, non-exclusive content provider surplus decreases if $a_1 > 2\delta v_2$, and platform profits increase if a_1 is high enough such that (21) holds. We show configurations of the parameters $\{r_1, a_1, r_2, v_0\}$ under which (21) is satisfied and study the impact on consumer surplus and welfare as the number of platforms increases.

Let $u^*_{1,n,\text{ENE}}$ be defined as consumers' utility in the exclusive–non-exclusive game with n platforms. Figure 1A shows simulated values for the difference in consumer' utility when the number of platforms increases from two to three in the exclusive–non-exclusive game, $u^*_{1,3,\text{ENE}} - u^*_{1,2,\text{ENE}}$. Figure 1A shows that consumers' surplus decreases as n goes from two to three platforms; that is, $u^*_{1,3,\text{ENE}} - u^*_{1,2,\text{ENE}} < 0$.[43] Figure 1B shows the total amount of content available in a platform in the exclusive–non-exclusive game decreases when n goes from two to three platforms (i.e., $n^*_{2,2} + n^{m*}_{2,2} > n^*_{2,3} + n^{m*}_{2,3}$). Then, in the exclusive–non-

[42]We do not have a formal proof of the change in consumer surplus when n goes from two to three platforms. However, our simulations show that consumers surplus decreases when the number of platforms increases from two to three (see Figure 1A).

[43]Note that in the exclusive-only game, consumer prices increase and the amount of exclusive content available to consumers decreases, which implies that $u^*_{1,3,\text{E}} < u^*_{1,2,\text{E}}$.

exclusive game, consumer prices increase (see Corollary 4(i)) and the amount of content available to consumers decreases (see Figure 1B), explaining why $u^*_{1,3,\text{ENE}} < u^*_{1,2,\text{ENE}}$.

Finally, let $W_{n,\text{ENE}}$ be the overall welfare in the exclusive–non-exclusive game with n platforms. Figure 1C shows that overall welfare decreases as n goes from two to three platforms; that is, $W_{3,\text{ENE}} - W_{2,\text{ENE}} < 0$. Note that, as n goes from two to three platforms, consumer and content provider surpluses decrease, these losses outweigh gains in platforms' profits, explaining why $W_{3,\text{ENE}} < W_{2,\text{ENE}}$.[44]

Figure 1A: Difference in Consumers' Utilities

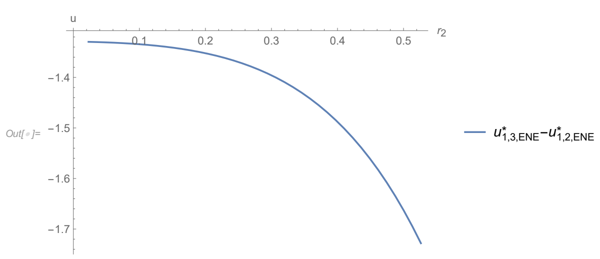
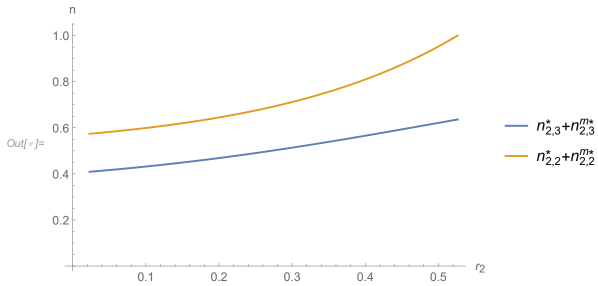


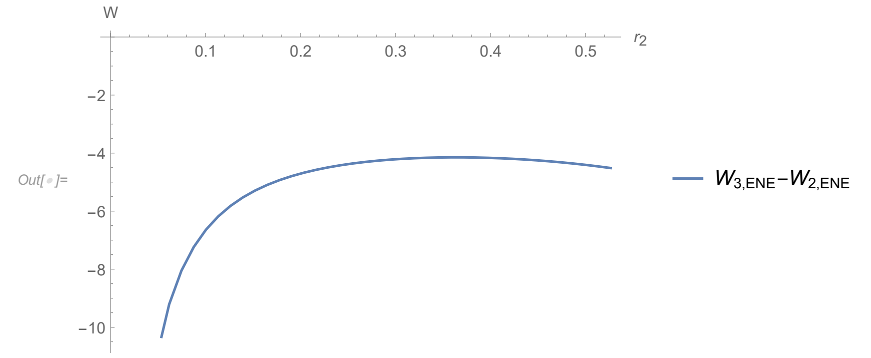
Figure 1B: Content Providers



Note: Figure 1A shows the change in consumers' utility as a function of r_2 when n goes from two to three platforms in the exclusive–non-exclusive game, $u^*_{1,3,\text{ENE}} - u^*_{1,2,\text{ENE}}$ is negative and decreasing in r_2. For this graph, we have fixed values of $\delta = 0.9$, $v_2 = 1$, $a_2 = 2$, and $a_1 = 2$. Figure 1B shows the total amount of content providers available in a platform as a function of r_2 when there are two or three platforms in the market.

[44] Note that Figures 1A-1B and Figure 1C use specific values of the parameters, $\delta = 0.9$, $v_2 = 1$, $r_1 = 0.4$, $a_2 = 2$, and $a_1 = 2$. However, the qualitative behaviors in these graphs (i.e., $u^*_{1,3,\text{ENE}} - u^*_{1,2,\text{ENE}} < 0$, $n^*_{2,2} + n^{m*}_{2,2} > n^*_{2,3} + n^{m*}_{2,3}$, and $W_{3,\text{ENE}} - W_{2,\text{ENE}} < 0$) remain valid for many other configurations of the parameters $\{a_1, a_2, r_1, \delta, v_2\}$. We were unable to find a configuration of these parameters for which the results in the Figures 1A-1B and Figure 1C did not hold.

Figure 1C: Difference in Welfare



Note: Figure 1C shows the difference in welfare, $W_{3,\mathrm{ENE}} - W_{2,\mathrm{ENE}}$, as function of r_2 (on the x-axis). For the simulation, we use the following parameters: $\delta = 0.9$, $v_2 = 1$, $a_2 = 2$, and $a_1 = 2$.

5.1 Deterring Entry

In this subsection, we explore the possibility of introducing or removing non-exclusive contracts as a way to deter entry. Suppose a market with two incumbent platforms facing one potential entrant. These platforms can jointly commit to offer exclusive-only or exclusive–non-exclusive contracts to deter entry.[45]

We study the following game: Suppose that the two incumbent platforms jointly commit to offer exclusive-only (E) or exclusive–non-exclusive prices (ENE). The potential entrant pays an entry cost, c, to enter the market. If the incumbent platforms offer exclusive-only (exclusive–non-exclusive) prices and the entrant enters the market, then each of the incumbent platforms receive $\pi_{3,\mathrm{E}}$ ($\pi_{3,\mathrm{ENE}}$), and the entrant receives $\pi_{3,\mathrm{E}} - c$ ($\pi_{3,\mathrm{ENE}} - c$); if the entrant platform does not enter the market, it receives 0, while each of the incumbent platforms receives $\pi_{2,\mathrm{E}}$ ($\pi_{2,\mathrm{ENE}}$). Note that, by Corollary 3, $\pi_{3,\mathrm{E}} < \pi_{3,\mathrm{ENE}}$.[46] The following proposition characterizes the equilibrium of the game.

Proposition 5. (i) *If $c \in (\pi_{3,\mathrm{E}}, \pi_{3,\mathrm{ENE}})$ and $\pi_{3,\mathrm{ENE}} < \pi_{2,\mathrm{E}}$, then the incumbent platforms offer exclusive-only contracts and the entrant does not enter;*

(ii) *if $c < \pi_{3,\mathrm{E}}$ or $c > \pi_{3,\mathrm{ENE}}$, the incumbent platforms offer exclusive–non-exclusive contracts, and the entrant enters the market only if $c < \pi_{3,\mathrm{E}}$.*

[45] Notice that if the two incumbents do not offer non-exclusive prices, then the entrant is effectively forced to offer exclusive prices only, since no content provider will be able to accept its non-exclusive price.

[46] In fact, it is not difficult to show that after the introduction of non-exclusive prices in the exponential distribution game with two platforms, $\pi_{2,\mathrm{E}} < \pi_{2,\mathrm{ENE}}$.

Proposition 5(i) shows that if $c \in (\pi_{3,\mathrm{E}}, \pi_{3,\mathrm{ENE}})$, the entrant's optimal strategy depends on the incumbents' decision. Using backward induction, Proposition 5(i) shows that the incumbent platforms deter entry by offering exclusive-only contracts if and only if $\pi_{3,\mathrm{ENE}} < \pi_{2,\mathrm{E}}$. That is, platforms will switch to offering exclusive contracts only if the profits in the exclusive-only game with two platforms are larger than the profits in the exclusive–non-exclusive game with three platforms.

Note that $\pi_{3,\mathrm{ENE}} < \pi_{2,\mathrm{E}}$ if and only if

$$2\left(a_1 + a_2\right) - \left(\frac{1}{r_1} + \frac{1}{r_2}\right) < h_2\left(v_0, r_2\right), \tag{22}$$

where $h_2\left(v_0, r_2\right) \equiv -\frac{1}{r_2}\frac{y^3\left(9y^5-15y^4+y^3+y^2-6y+16\right)}{1-y^4} < 0$. From (22) and the definition of h_2, it follows that a necessary condition for platforms to deter entry is that $2(a_1+a_2) < \frac{1}{r_1}+\frac{1}{r_2}$; that is, marginal network benefits must be small relative to the expected random utility components of users.

From Proposition 5, it follows that platforms cannot deter entry by offering exclusive–non-exclusive contracts. If the incumbent platforms choose to offer exclusive–non-exclusive contracts, the entrant's decision depends on the value of c. Proposition 5(ii) shows that if $c < \pi_{3,\mathrm{E}}$, then it enters the market; if $c > \pi_{3,\mathrm{ENE}}$, then it does not enter the market. In these two cases, incumbent platforms will always offer exclusive–non-exclusive prices, since $\pi_{n,\mathrm{E}} < \pi_{n,\mathrm{ENE}}$ for $n = 2, 3$. In conclusion, offering exclusive-non-exclusive contracts does not help platforms to deter entry.

Finally, in the following corollary, we compare both games—the exclusive-only and exclusive–non-exclusive game—when n increases from two to three platforms.

Corollary 5. *Assume $\frac{1}{r_2} > \frac{1}{2}a_2 + \delta v_2$. If the number of platforms, n, increases from two to three: (i) platforms are better off in the exclusive–non-exclusive game than in the exclusive-only game if and only if $h(v_0, r_2) < 0$; (ii) exclusive content providers are worse off in the exclusive–non-exclusive game than in the exclusive-only game.*

If platforms' profits increase when n increases from two to three in both games and $h(v_0, r_2) < 0$ (> 0), then platforms in the exclusive–non-exclusive game will gain more (less) than those in the exclusive-only game. Similarly, if platforms' profits decrease when the number of platforms increases, then, if $h(v_0, r_2) < 0$ (> 0), platforms in the exclusive–non-exclusive game will lose less (more) than those in the exclusive-only game.

Intuitively, when the number of platforms increases in the exclusive–non-exclusive game, the exclusive content provider price increases by $\frac{a_1}{2} + \frac{y^4}{r_2(1-y^4)}$; whereas in the exclusive-only game, the price increases by $\frac{a_1}{2}$. Thus, $\frac{y^4}{r_2(1-y^4)}$ measures the greater loss of the exclusive content providers in the exclusive–non-exclusive game.[47] Note that when platforms are allowed to offer both exclusive and non-exclusive prices, they have more tools to extract surplus from content providers.

6 Discussion

In this section, we discuss examples of two-sided platforms and how they fit with our model prediction. First, we explore examples where exclusive and non-exclusive content providers are compensated differently, focusing on the difference in payment received by exclusive versus non-exclusive content providers. Then, we discuss examples related to platform entry, focusing on price changes by incumbent platforms and changes in content availability to consumers.

Exclusive–non-exclusive markets. Live streaming on *Twitch.tv* is an example where content providers self-select into exclusive or non-exclusive contracts. Content providers can stream on *Twitch.tv* at different participation levels. A Basic user can stream on the platform but does not receive any ad revenue. An Affiliate receives a share of the ad revenue, but its streaming content is bounded by a timed-exclusivity clause prohibiting the content from being broadcast on other platforms within 24 hours. Content providers that wish to earn ad revenue from participation on *Twitch.tv* must bind themselves to the exclusivity agreement. This exclusivity agreement echoes our exclusive–non-exclusive model with negative content provider prices in equilibrium. While most individual streamers only live-stream on *Twitch.tv*, other content providers such as news outlets (e.g., *FOX*, *ABC*) and political campaigns (e.g., election candidate rallies) often live-stream simultaneously on *Youtube*, *Facebook*, and *Twitch.tv*. These content providers are not interested in the platform payments; they are often looking for benefits separate from streaming (e.g., political donations, advertisement contracts unrelated to the platforms, or merchandise sales). This observation is consistent with our assumption that multi-homing content providers join all platforms.

Similarly, ridesharing services like *Uber* and *Lyft* offer special benefits to drivers who act as exclusive contractors. To be eligible for these benefits, both platforms have (high) cutoffs (in terms of the number of rides) that make drivers exclusively work with one firm, whereas consumers multi-home across the two platforms. Finally, cable companies are a good example where consumers usually single-home, while content providers multi-home to multiple (some of them to all) cable companies. The credit card market is a common example of platforms that match buyers and sellers. Sellers rarely receive

[47] Note that $\frac{y^4}{r_2(1-y^4)}$ is increasing in y, the proportion of multi-homers in the exclusive–non-exclusive game; thus, the greater this proportion is, the greater the loss for exclusive content providers.

payments exclusively through only one credit card issuer, and it is practically impossible for credit card issuers to limit the number of credit cards in the consumer's wallet (buyer side). Exclusivity is ensured through the conditions under which cardholders can use the benefits offered. For example, credit card issuers often offer discounted travel and rental car insurance on the condition that the cardholder pays the airline ticket or car rental cost exclusively with the issuer's card.[48]

In the examples above, users are allowed to self-select into exclusive or non-exclusive contracts. In either case, the self-selected exclusive users receive higher benefits from the platform than non-exclusive users. Our model predicts that non-exclusive users and platforms are better off when non-exclusive contracts are introduced, and exclusive users are worse off.

eSports events are another example of multi-homing content providers that live-stream their content. Unlike the self-selected multi-homing content providers we discussed above, these content providers usually negotiate a deal with each platform. Carroni et al. (2020) show that big content providers that are vertically integrated with a platform are more likely to prefer multi-homing than non-integrated big content providers. Karle et al. (2020) consider a coordination game where multiple big discrete content providers decide between single-homing or multi-homing. Platforms can sustain high prices when content provider competition is intense because content providers are eager to avoid each other.

Platform Entry. *Disney Plus*'s video streaming platform is an example of platform entry. Consistent with our model's prediction, rival video streaming platform *Netflix* announced a price increase soon after Disney entered the market.[49] The increase in price is a result of *Netflix* being able to attract fewer users from existing rivals (e.g., *Hulu* and *Amazon Prime Video*), since the entry of *Disney Plus* cannibalized the existing user base and created a new segment of loyal users who are unlikely to be attracted by other platforms lowering their prices.

Disney's entry into the video streaming market raised concerns among users of existing video streaming platforms: Disney content that was previously streamed on a variety on platforms would no longer be available on *Netflix* or other streaming services. Our model shows that platform entry will splinter the existing user base into smaller fragments, resulting in smaller benefits for content providers, while prices increase. The surplus loss

[48] Insurance is not a direct payment, but a form of tying goods. Amelio and Jullien (2012) show that tying goods can act as a form of substitute payment and improve welfare in platform markets.

[49] *Disney Plus* was announced in August 2017, and *Netflix* announced a price increase in October 2017. *Netflix*'s price increase was the second price increase in 18 months; the timing and frequency suggests that it was related to *Disney*'s entry (https://www.nytimes.com/2017/08/09/business/media/with-disneys-move-to-streaming-a-new-era-begins.html; http://www.marketwatch.com/story/did-netflix-choose-a-perfect-time-to-hike-subscription-costs-2017-10-06).

is partially alleviated by having *Disney* better serve its loyal users.[50]

We explore the possibility of introducing or removing non-exclusive contracts as a way to deter entry. Previous literature has studied the impact of exclusive contracts on platform choice. Lee (2013) estimated the impact of exclusive titles on the adoption of sixth-generation video game consoles, concluding that exclusivity promotes entry of smaller platforms but does not benefit the larger incumbent. Intuitively, new entrants benefit from exclusive contracts by providing novel content that incumbents do not have, which attracts consumers and strengthens their user base. *Disney Plus*'s content *The Mandalorian* piqued interest in the new platform, helping it to build a consumer base. Corts and Lederman (2009) and Nair (2007) estimate the impact of exclusive and non-exclusive titles on console adoption. Corts and Lederman (2009) report that 40% of total video game titles to be non-exclusively available on multiple consoles, and Prieger and Hu (2008) suggest that "PlayStation 2 and Xbox garner most of their revenue from non-exclusive titles." Our paper complements the discussion on exclusive-contracts by showing that platforms cannot deter entry by offering exclusive–non-exclusive contracts. In certain circumstances, platforms will jointly deviate and offer exclusive contracts only, in order to deter entry.

7 Conclusion

In this paper, we study how the availability of both exclusive and non-exclusive contracts affects two-sided market outcomes and how the presence of these contracts impacts the share of exclusive and non-exclusive content available on platforms.

We consider a model with n symmetric platforms serving a two-sided market with two types of users: *consumers* and *content providers*. We study the subgame perfect Nash equilibrium of the two-stage problem and provide two examples where a unique symmetric subgame perfect Nash equilibrium exists with three platforms (when the idiosyncratic preferences are exponentially and uniformly distributed). We show that the introduction of non-exclusive contracts softens the competition for content providers between platforms, as they have more tools to extract content surplus; that is, they have a

[50]Competition between *Disney* and *Netflix* is different from our model in one important way: both platforms are major content providers to their own streaming services. Hagiu and Lee (2011), Carrillo and Tan (2018), Carroni et al. (2020), and Karle et al. (2020) consider the possibility of platforms making content production decisions by purchasing content providers. Athey et al. (2016) studied platforms' ability to produce content; however users were exogenously determined to be single-homing or multi-homing, which limited the model's ability to understand the impact on availability of multi-homing content. Carroni et al. (2020) focused on the incentive of superstar content providers to multi-home, but did not explore superstars' impact on fringe content providers' decision to multi-home. The question of how vertically integrated content providers would impact the availability of multi-homing fringe content providers is an interesting direction in which to extend the current research.

new margin to extract content providers' surplus—the exclusive–non-exclusive margin—in addition to the exclusive-exclusive margin. Consumers pay the same price as that in the exclusive-only game, but gain access to new content provided by multi-homing content providers, increasing their surplus. Content providers remaining as single-homers are gaining access to the same mass of consumers, but being paid less, resulting in a loss. Content providers that become multi-homers now receive a payment from each platform and receive multiple realizations of positive idiosyncratic preferences. Platforms' profits increase by two means: the higher price being charged to exclusive content providers, and the the gains from charging more to non-exclusive content providers than to exclusive content providers. Finally, we show that welfare increases when the share of multi-homing content providers is small or close to one.

We show that when the number of platforms increases from two to three in the exclusive–non-exclusive game, equilibrium prices increase. As the number of platforms increases, exclusive content providers are worse off due to the higher price and the smaller market share. Non-exclusive content providers are worse off if the discounted base utility is smaller than consumers' marginal network benefit. We show that platforms' profits increase if the network benefit on the consumers' side is high, so that the price effects dominate the market share effects. Thus, having more competing platforms in the market is not necessarily consumer welfare enhancing in the exclusive–non-exclusive game.

Finally, we show that platforms cannot deter entry by offering exclusive–non-exclusive contracts. In certain circumstances, platforms will jointly deviate from the exclusive–non-exclusive game and offer exclusive prices only, in order to deter entry. These results contribute to analyses with important antitrust applications.

One plausible extension is to consider the horizontal quality of the content: The literature has yet to address what happens when consumers have preferences for different content providers on the other side. Intuitively, this feature is expected to dampen the impact of fragmentation by offering the benefits of more "filtered" results, such as content providers easily reaching their target audiences and dating site participants easily meeting people with similar backgrounds. A second extension is to consider the effects on competition when platforms integrate with content providers. Our non-tipping models provided insights on availability of non-exclusive contents, which tipping models are unable to analyze.

Appendix

A Proofs

Proof of Theorem 1. We assume full market coverage and Assumption 1. Suppose that there exists a unique solution β^* of the equation (16) and that $J(\beta^*) \neq 0$, where J is the Jacobian associated with equations (13). We show that there exists a symmetric subgame perfect Nash equilibrium $(\boldsymbol{n}^*,\ \boldsymbol{p}^*)$, where all platforms charge prices $\boldsymbol{p}^* = (p_1^*, p_2^*, p_{2m}^*)$ given by (17), and the equilibrium market shares are given by $\boldsymbol{n}^* = \left(n_1^* = \frac{1}{n}, n_2^* = \frac{1}{n}(1-x), n_2^{m*} = x\right)$. The proof has two steps:

(i) We show that given a deviation in quantities $\boldsymbol{n}^1$ from the proposed symmetric equilibrium (SE), there is a corresponding vector of prices $\boldsymbol{p}^1$ such that $(\boldsymbol{n}^1, \boldsymbol{p}^1)$ is a participative equilibrium (PE).

(ii) We derive the profit function of a platform that deviates from the proposed SE and show that this deviation is not profitable.

(i) Assume that platform $i = 1$ deviates from the proposed SE by choosing quantities $\boldsymbol{n}^1 = (n_1, n_2, n_2^m)$. Platforms that do not deviate from the SE charge prices (p_1^*, p_2^*, p_{2m}^*) given by (17). We show that there is a vector of prices $\boldsymbol{p}^1$ such that $(\boldsymbol{n}^1, \boldsymbol{p}^1)$ is a PE. Note that $\boldsymbol{n}^1$ is a PE of $\boldsymbol{p}^1$ if and only if $\boldsymbol{n}^1$ solves

$$\begin{cases} n_1 & = Q_1^1\left(u_1^1, u_1^*, \ldots, u_1^*, 0\right) \\ n_2 & = Q_2^1\left(u_2^1, u_2^*, \ldots, u_2^*, u_2^m\right) \\ n_2^m & = Q_2^m\left(u_2^1, u_2^*, \ldots, u_2^*, u_2^m\right), \end{cases} \quad \text{(A.1)} \quad \text{and} \quad \begin{cases} \frac{1-n_1}{n-1} & = Q_1^j\left(u_1^1, u_1^*, \ldots, u_1^*, 0\right) \\ \frac{1-n_2-n_2^m}{n-1} & = Q_2^j\left(u_2^1, u_2^*, \ldots, u_2^*, u_2^m\right) \\ \text{for each} & j \in \{2, \cdots, n\}. \end{cases} \quad \text{(A.2)}$$

Quantity u_1^1 is the deterministic utility component of a consumer who joins platform 1, and u_2^1 the deterministic utility component of a content provider that joins platform 1. The quantities u_1^* and u_2^* are the deterministic utility components of consumers and content providers, respectively, that join one of the $n-1$ non-deviating platforms. The quantity u_2^m is the deterministic utility component of content providers multi-homing to all platforms. These terms are defined, respectively, as

$$
\begin{aligned}
u_1^1 &= v_1 + \phi_1\left(n_2, n_2^m\right) - p_1, \\
u_2^1 &= v_2 + \phi_2\left(n_1\right) - p_2, \\
u_1^* &= v_1 + \phi_1\left(\frac{1 - n_2 - n_2^m}{n-1}, n_2^m\right) - p_1^*, \\
u_2^* &= v_2 + \phi_2\left(\frac{1-n_1}{n-1}\right) - p_2^*, \text{ and} \\
u_2^m &= v_2 + v_2\left(n-1\right)\delta + \phi_m\left(n_1, \frac{1-n_1}{n-1} \cdot \mathbf{1}_{n-1}\right) - p_{2m} - \left(n-1\right)p_{2m}^*.
\end{aligned}
\tag{A.3}
$$

We define the random variables (RVs)

$$
\begin{aligned}
X_k &\equiv \varepsilon_k^1 - \max_{j \neq 1,\ j \in \mathcal{N}} \left(\varepsilon_k^j\right), \quad \text{for } k \in \{1,2\}, \\
Y &\equiv -\sum_{j=2}^{n} \varepsilon_2^j, \text{ and} \\
Z &\equiv \boldsymbol{\varepsilon}_2 \cdot \mathbf{1}_n - \max_{j \neq 1,\ j \in \mathcal{N}} \left(\varepsilon_2^j\right).
\end{aligned}
\tag{A.4}
$$

For any x_0, y_0, z_0 in $\mathbb{R}$, we define the distributions

$$
\begin{aligned}
H_1(x_0) &\equiv \mathbb{P}(X_1 \leq x_0), \\
F^1\left(x_0, y_0\right) &\equiv \mathbb{P}\left(X_2 > x_0, Y > y_0\right), \text{ and} \\
F^2\left(z_0, y_0\right) &\equiv \mathbb{P}\left(Z > z_0, Y < y_0\right).
\end{aligned}
\tag{A.5}
$$

Let $x(\beta^*) \equiv F^2(-\beta^*, \beta^*)$. From (A.1), (9)-(11), and (A.4)-(A.5), the market shares of the deviating platform $\boldsymbol{n}^1$ satisfy

$$
\begin{aligned}
n_1 &= \mathbb{P}\left(\varepsilon_1^1 - \max_{j \neq 1,\ j \in \mathcal{N}} \left(\varepsilon_1^j\right) > u_1^* - u_1^1\right) \\
&= 1 - H_1\left(u_1^* - u_1^1\right), \\
n_2 &= \mathbb{P}\left(X_2 > u_2^* - u_2^1, Y > u_2^m - u_2^1\right) \\
&= F^1(u_2^* - u_2^1, u_2^m - u_2^1), \text{ and} \\
n_2^m &= \mathbb{P}\left(Z > u_2^* - u_2^m, Y < u_2^m - u_2^1\right) \\
&= F^2(u_2^* - u_2^m, u_2^m - u_2^1).
\end{aligned}
\tag{A.6}
$$

Note that (A.6) can be rewritten as

$$\begin{aligned} n_1 - 1 + H_1\left(u_1^* - u_1^1\right) &= 0, \\ F^1(u_2^* - u_2^1, u_2^m - u_2^1) - n_2 &= 0, \text{ and} \\ F^2(u_2^* - u_2^m, u_2^m - u_2^1) - n_2^m &= 0. \end{aligned} \tag{A.7}$$

From (A.3), we know that the quantities $u_1^* - u_1^1$, $u_2^* - u_2^1$, $u_2^m - u_2^1$, and $u_2^* - u_2^m$ are functions of $(\boldsymbol{n}^1, \boldsymbol{p}^1) = (n_1, n_2, n_2^m, p_1, p_2, p_{2m})$. Thus, equations in (A.7) constitute a system of non-linear equations in the variables $(\boldsymbol{n}^1, \boldsymbol{p}^1)$.

Claim: (A.7) implicitly determines the inverse demand function $\boldsymbol{n}^1 \mapsto \boldsymbol{p}^1(\boldsymbol{n}^1) = (p_1(\boldsymbol{n}^1), p_2(\boldsymbol{n}^1), p_{2m}(\boldsymbol{n}^1))$, where $\boldsymbol{p}^1(\boldsymbol{n}^1)$ is such that $(\boldsymbol{n}^1, \boldsymbol{p}^1(\boldsymbol{n}^1))$ is a PE.

Proof of the Claim: We use the Implicit Function Theorem (IFT) to show that there exists a neighborhood of $\boldsymbol{n}^*$, and an inverse demand function $\boldsymbol{p}^1 = (p_1(\boldsymbol{n}^1), p_2(\boldsymbol{n}^1), p_{2m}(\boldsymbol{n}^1))$ that solves (A.7) for each $\boldsymbol{n}^1$ in that neighborhood.

In order to apply the IFT, we verify the following two conditions:

(a) (A.7) has a solution at $(\boldsymbol{n}, \boldsymbol{p}) = (\boldsymbol{n}^*, \boldsymbol{p}^*)$ where $\boldsymbol{p}^*$ is given by (17).

(b) The Jacobian J of (A.7) with respect to the variables in $\boldsymbol{p}^1 = (p_1, p_2, p_{2m})$ at $(\boldsymbol{n}^*, \boldsymbol{p}^*)$ is non-zero.

(a) At $(\boldsymbol{n}^*,\boldsymbol{p}^*) = (\frac{1}{n}, \frac{1}{n}\left[1 - x\left(\beta^*\right)\right], x\left(\beta^*\right), p_1^*, p_2^*, p_{2m}^*)$, (A.7) reduces to

$$\begin{aligned} \frac{1}{n} - 1 + H_1(0) &= 0, \\ F^1(0, \beta^*) - \frac{1}{n}(1 - x(\beta^*)) &= 0, \text{ and} \\ F^2(-\beta^*, \beta^*) - x(\beta^*) &= 0, \end{aligned}$$

where $H_1(0) = \frac{n-1}{n}$. By definition, $x(\beta^*) = F^2(-\beta^*, \beta^*)$, and by the full market coverage assumption, $F^1(0, \beta^*) = \frac{1}{n}(1 - x(\beta^*))$. Thus, (A.7) has a solution at $(\boldsymbol{n}^*, \boldsymbol{p}^*)$.

(b) The Jacobian (A.7) is

$$J(\beta^*) = \begin{vmatrix} \frac{\partial H_1}{\partial p_1} & \frac{\partial H_1}{\partial p_2} & \frac{\partial H_1}{\partial p_2^m} \\ \frac{\partial F^1}{\partial p_1} & \frac{\partial F^1}{\partial p_2} & \frac{\partial F^1}{\partial p_{2m}} \\ \frac{\partial F^2}{\partial p_1} & \frac{\partial F^2}{\partial p_2} & \frac{\partial F^2}{\partial p_{2m}} \end{vmatrix}_{(\boldsymbol{n},\boldsymbol{p})=(\boldsymbol{n}^*,\boldsymbol{p}^*)} = \begin{vmatrix} h_1 & 0 & 0 \\ 0 & F_1^1 + F_2^1 & -F_2^1 \\ 0 & F_2^2 & F_1^2 - F_2^2 \end{vmatrix}_{(\boldsymbol{n},\boldsymbol{p})=(\boldsymbol{n}^*,\boldsymbol{p}^*)},$$

where h_1 is the derivative of the distribution H_1 given by (A.5), and F_k^i represents the partial derivative of F^i in the k-coordinate. From (15), β^* is

$$\beta^* = u_2^{m*} - u_2^* = v_2 (n-1)\delta + \phi_m\left(\frac{1}{n}\cdot \mathbf{1}_n\right) - \phi_2\left(\frac{1}{n}\right) + p_2^* - np_{2m}^*.$$

Thus the Jacobian $J(\beta^*)$ can expressed as

$$\begin{aligned} J(\beta^*) &= h_1(0)\underbrace{\left[F_1^1(0,\beta^*)\left(F_1^2(-\beta^*,\beta^*) - F_2^2(-\beta^*,\beta^*)\right) + F_2^1(0,\beta^*)F_1^2(-\beta^*,\beta^*)\right]}_{\equiv D(\beta^*)} \\ &= h_1(0)D(\beta^*). \end{aligned} \tag{A.8}$$

By assumption, $J(\beta^*) \neq 0$. From the IFT, there exists a neighborhood of $\boldsymbol{n}^*$ and an inverse demand function $\boldsymbol{p}^1 = (p_1(\boldsymbol{n}^1), p_2(\boldsymbol{n}^1), p_{2m}(\boldsymbol{n}^1))$ that solves (A.7) for each $\boldsymbol{n}^1$ in that neighborhood.

(ii) The profit function of the deviating platform is

$$\pi_1(\boldsymbol{n}^1) = n_1 \cdot p_1(\boldsymbol{n}^1) + n_2 \cdot p_2(\boldsymbol{n}^1) + n_2^m \cdot p_{2m}(\boldsymbol{n}^1). \tag{A.9}$$

We show that any deviation from the SE is not profitable. Note that any stationary point of $\pi^1(\boldsymbol{n}^1)$ satisfies

$$\begin{aligned} \frac{\partial \pi^1}{\partial n_1} &= p_1 + \frac{\partial p_1}{\partial n_1}n_1 + \frac{\partial p_2}{\partial n_1}n_2 + \frac{\partial p_{2m}}{\partial n_1}n_2^m = 0, \\ \frac{\partial \pi^1}{\partial n_2} &= p_2 + \frac{\partial p_2}{\partial n_2}n_2 + \frac{\partial p_1}{\partial n_2}n_1 + \frac{\partial p_{2m}}{\partial n_2}n_2^m = 0, \text{ and} \\ \frac{\partial \pi^1}{\partial n_2^m} &= p_{2m} + \frac{\partial p_{2m}}{\partial n_2^m}n_2^m + \frac{\partial p_1}{\partial n_2^m}n_1 + \frac{\partial p_2}{\partial n_2^m}n_2 = 0, \end{aligned} \tag{A.10}$$

where $\frac{\partial \boldsymbol{p}^1}{\partial \boldsymbol{n}^1} = \frac{\partial(p_1,p_2,p_{2m})}{\partial(n_1,n_2,n_2^m)}$ at $\boldsymbol{n}^*$ is given by

$$\begin{bmatrix} \frac{\partial p_1}{\partial n_1} \\ \frac{\partial p_2}{\partial n_1} \\ \frac{\partial p_{2m}}{\partial n_1} \end{bmatrix} = \begin{bmatrix} -\frac{1}{h_1(0)} \\ \frac{n}{n-1}\phi_2'\left(\frac{1}{n}\right) \\ \frac{1}{n-1}\phi_2'\left(\frac{1}{n}\right) \end{bmatrix},$$
$$\begin{bmatrix} \frac{\partial p_1}{\partial n_2} \\ \frac{\partial p_2}{\partial n_2} \\ \frac{\partial p_{2m}}{\partial n_2} \end{bmatrix} = \frac{1}{D(\beta^*)}\begin{bmatrix} D(\beta^*)\frac{n}{n-1}\frac{\partial \phi_1}{\partial n_2^i}\left(\frac{1}{n}(1-x),x\right) \\ F_1^2(-\beta^*,\beta^*) - F_2^2(-\beta^*,\beta^*) \\ -F_2^2(-\beta^*,\beta^*) \end{bmatrix}, \text{ and} \tag{A.11}$$
$$\begin{bmatrix} \frac{\partial p_1}{\partial n_2^m} \\ \frac{\partial p_2}{\partial n_2^m}, \\ \frac{\partial p_{2m}}{\partial n_2^m} \end{bmatrix} = \frac{1}{D(\beta^*)}\begin{bmatrix} D(\beta^*)\frac{1}{n-1}\frac{\partial \phi_1}{\partial n_2^i}\left(\frac{1}{n}(1-x),x\right) \\ F_2^1(0,\beta^*) \\ F_1^1(0,\beta^*) + F_2^1(0,\beta^*) \end{bmatrix}.$$

Note that $D(\beta^*) \neq 0$, since $J(\beta^*) \neq 0$. After imposing symmetry and plugging equations (A.11) into (A.10), p_1^* is given by

$$p_1^* - \frac{1 - H_1(0)}{h_1(0)} + \frac{1}{n-1}\phi_2'\left(\frac{1}{n}\right) = 0, \tag{A.12}$$

and $\{p_2^*, p_{2m}^*\}$ are given by

$$\begin{aligned} &p_2^* + \frac{1}{n-1}\frac{\partial \phi_1}{\partial n_2^i}\left(\frac{1}{n}(1-x),x\right) \\ &+ \frac{1}{D(\beta^*)}\left[\frac{1}{n}\left(F_1^2(-\beta^*,\beta^*) - F_2^2(-\beta^*,\beta^*)\right)(1-x) - F_2^2(-\beta^*,\beta^*)\,x\right] = 0, \text{ and} \end{aligned} \tag{A.13}$$

$$\begin{aligned} &p_{2m}^* + \frac{1}{n(n-1)}\frac{\partial \phi_1}{\partial n_2^i}\left(\frac{1}{n}(1-x),x\right) \\ &+ \frac{1}{D(\beta^*)}\left[\left(F_1^1(0,\beta^*) + F_2^1(0,\beta^*)\right)x + \frac{1}{n}(1-x)\,F_2^1(0,\beta^*)\right] = 0. \end{aligned} \tag{A.14}$$

We define the functions $M_2 : \mathbb{R} \longrightarrow \mathbb{R}$, and $M_{2m} : \mathbb{R} \longrightarrow \mathbb{R}$ as

$$\begin{aligned} M_2(\beta) &\equiv -\frac{1}{D(\beta)}\left[\frac{1}{n}\left(F_1^2(-\beta,\beta) - F_2^2(-\beta,\beta)\right)(1-x(\beta)) - F_2^2(-\beta,\beta)\,x(\beta)\right], \\ M_{2m}(\beta) &\equiv -\frac{1}{D(\beta)}\left[\left(F_1^1(0,\beta) + F_2^1(0,\beta)\right)x(\beta) + \frac{1}{n}(1-x(\beta))\,F_2^1(0,\beta)\right]. \end{aligned} \tag{A.15}$$

It follows that $\boldsymbol{p}^*$ is given by

$$\begin{aligned} p_1^* &= \frac{1 - H_1(0)}{h_1(0)} - \frac{1}{n-1}\phi_2'\left(\frac{1}{n}\right), \\ p_2^* &= M_2(\beta^*) - \frac{1}{n-1}\frac{\partial \phi_1}{\partial n_2^i}\left(\frac{1}{n}(1-x), x\right), \text{ and} \\ p_{2m}^* &= M_{2m}(\beta^*) - \frac{1}{n(n-1)}\frac{\partial \phi_1}{\partial n_2^i}\left(\frac{1}{n}(1-x), x\right). \end{aligned} \tag{A.16}$$

From Assumption 1, $\boldsymbol{n}^*$ is the unique stationary point of (A.10). It follows that any deviation from the SE is not profitable, which concludes the proof of the theorem.

Proof of Proposition 2. We prove this proposition by contradiction. We assume that $\beta^* \geq 0$ and show that the equilibrium is not interior. Suppose that $\beta^* \geq 0$. From (A.6),

$$n_2^* = \mathbb{P}\left(X_2 \geq 0, -\sum_{j=2}^{n} \varepsilon_2^j \geq \beta^*\right).$$

If the support of the distribution of the function of the idiosyncratic preferences $\{\varepsilon_2^i\}_{i=1}^n$ is positive, then

$$n_2^* \leq \mathbb{P}\left(-\sum_{j=2}^{n} \varepsilon_2^j \geq \beta^*\right) \underbrace{\leq}_{\beta^* \geq 0} \mathbb{P}\left(-\sum_{j=2}^{n} \varepsilon_2^j \geq 0\right) = \mathbb{P}\left(\sum_{j=2}^{n} \varepsilon_2^j \leq 0\right) = 0.$$

It follows that $n_2^* = 0$ and $n_2^{m*} = 1$, so that the equilibrium is not interior.

Lemma 1. *Assume linear network benefit functions. For $s, r \in \{1, 2, 3\}$, d_{sr} denotes the (s, r)-entry of the Hessian matrix of the profit function, $D^2\pi^1$. Suppose that*

(a) H_1 (defined by A.5) is log-concave;

(b) $d_{11}d_{22} > \left(\frac{n}{n-1}\right)^2 (a_1 + a_2)^2$;

(b) $d_{11}\left(d_{22}d_{33} - d_{23}^2\right) < \left(\frac{n}{n-1}\right)^2 (a_1 + a_2)^2 \left(d_{33} - \frac{2}{n}d_{23} + \frac{1}{n^2}d_{22}\right)$.

Then, Assumption 1 is satisfied.

Proof of Lemma 1. Note that Lemma 1 provides simpler conditions—on the distributions of the idiosyncratic preferences—which imply Assumption 1. Recall that π^1 is

$$\pi^1(\boldsymbol{n}^1) = n_1 \cdot p_1(\boldsymbol{n}^1) + n_2 \cdot p_2(\boldsymbol{n}^1) + n_2^m \cdot p_{2m}(\boldsymbol{n}^1), \tag{A.17}$$

where $\boldsymbol{p}^1(\boldsymbol{n}^1) = (p_1(\boldsymbol{n}^1), p_2(\boldsymbol{n}^1), p_{2m}(\boldsymbol{n}^1))$ is defined by (A.7). For $l \in \{1, 2, 2m\}$ and $k \in \{1, 2\}$, let

$$(p_l)_k \equiv \frac{\partial p_l}{\partial n_k} \quad \text{and} \quad (p_l)_3 \equiv \frac{\partial p_l}{\partial n_2^m}.$$

Similarly, for $k, j \in \{1, 2\}$, let $(p_l)_{kj} \equiv \frac{\partial^2 p_l}{\partial n_k \partial n_j}$. Suppose that $\phi_1(n_2^i, n_2^m) = a_1(n_2^i + n_2^m)$, $\phi_2(n_1^i) = a_2 n_1^i$, and $\phi_m(n_1^1, n_1^2, n_1^3) = a_2(n_1^1 + n_1^2 + n_1^3)$, where a_1, a_2 are positive constants. From (A.11) and (A.17),

$$D^2\pi^1 = \begin{bmatrix} 2(p_1)_1 + n_1(p_1)_{11} & \frac{n}{n-1}(a_1+a_2) & \frac{1}{n-1}(a_1+a_2) \\ \frac{n}{n-1}(a_1+a_2) & 2(p_2)_2 + n_2(p_2)_{22} + n_2^m(p_{2m})_{22} & (p_2)_3 + n_2(p_2)_{23} + (p_{2m})_2 + n_2^m(p_{2m})_{23} \\ \frac{1}{n-1}(a_1+a_2) & (p_2)_3 + n_2(p_2)_{23} + (p_{2m})_2 + n_2^m(p_{2m})_{23} & n_2(p_2)_{33} + 2(p_{2m})_3 + n_2^m(p_{2m})_{33} \end{bmatrix}. \tag{A.18}$$

Note that $D^2\pi^1$ can be expressed as

$$D^2\pi^1 = \begin{bmatrix} d_{11} & \frac{n}{n-1}(a_1+a_2) & \frac{1}{n-1}(a_1+a_2) \\ \frac{n}{n-1}(a_1+a_2) & d_{22} & d_{23} \\ \frac{1}{n-1}(a_1+a_2) & d_{23} & d_{33} \end{bmatrix}. \tag{A.19}$$

From (A.18) and (A.19), $D^2\pi^1$ is negative definite if and only if

$$\begin{aligned} &\text{(a) } d_{11} < 0, \\ &\text{(b) } d_{11}d_{22} > \left(\frac{n}{n-1}\right)^2 (a_1+a_2)^2, \text{ and} \\ &\text{(c) } d_{11}\left(d_{22}d_{33} - d_{23}^2\right) < \left(\frac{n}{n-1}\right)^2 (a_1+a_2)^2 \left(d_{33} - \frac{2}{n}d_{23} + \frac{1}{n^2}d_{22}\right). \end{aligned} \tag{A.20}$$

Note that if H_1 is log-concave, then (a) always holds true. Thus, if H_1 is log-concave, and (b) and (c) hold true, then Assumption 1 is satisfied.

Proof of Proposition 3. The proof has two steps: (i) We show that a symmetric equilibrium (SE) exists; (ii) We show that the second-order conditions are satisfied.

(i) We show that there exists a unique solution β^* of the equation (16) and that $J(\beta^*) \neq 0$ (defined by A.8). Then, from Theorem 1, the existence of the SE follows.

From (A.4), $X_k = \varepsilon_k^1 - \max\{\varepsilon_k^2, \varepsilon_k^3\}$ for $k \in \{1, 2\}$, $Y = -\varepsilon_2^2 - \varepsilon_2^3$, and $Z = \boldsymbol{\varepsilon_2} \cdot \mathbf{1}_3 -$

$\max\{\varepsilon_2^2, \varepsilon_2^3\}$. From (A.5), the distributions H^1, F^1, and F^2 are

$$H_1(x_0) = \mathbb{P}(X_1 \leq x_0) = \begin{cases} e^{r_1 x_0} - \frac{1}{3}e^{2r_1 x_0} & x_0 < 0 \\ 1 - \frac{e^{-r_1 x_0}}{3} & x_0 \geq 0 \end{cases}, \tag{A.21}$$

$$F^1(x_0, y_0) = \mathbb{P}(X_2 > x_0, Y > y_0) = \begin{cases} \frac{1}{3}e^{-r_2 x_0}\left(-4e^{\frac{3r_2 y_0}{2}} + 3e^{2r_2 y_0} + 1\right) & x_0 \geq 0 \wedge y_0 < 0 \\ 0 & y_0 \geq 0 \end{cases}, \tag{A.22}$$

and

$$\begin{aligned} F^2(z_0, y_0) &= \mathbb{P}(Z > z_0, Y < y_0) \\ &= \begin{cases} e^{-2r_2 z_0}\left(-2e^{r_2(y_0+z_0)} + 4e^{\frac{1}{2}r_2(y_0+2z_0)} - 1\right) & y_0 < 0 \wedge y_0 + 2z_0 > 0 \\ 1 & (y_0 = 0 \wedge z_0 \leq 0) \vee (y_0 > 0 \wedge z_0 < 0) \end{cases}. \end{aligned} \tag{A.23}$$

Suppose that if a solution β^* of equation (16) exists, it must be the case that $\beta^* < 0$.[51] We verify this ex-post and study the other case ($\beta^* \geq 0$) at the end of the proof. Suppose that $\beta < 0$, from (A.23), $x(\beta) = F^2(-\beta, \beta) = e^{2r_2\beta}\left(4e^{-\frac{1}{2}r_2\beta} - 3\right)$. Let $y \equiv e^{\frac{\beta r_2}{2}}$, then $x(\beta) = 4y^3 - 3y^4$. From (A.15), (A.22), and (A.23) we get that

$$\begin{aligned} M_2(\beta) &= \frac{1}{r_2(1-y^4)}, \text{ and} \\ M_{2m}(\beta) &= \frac{6 + y + y^2 + y^3 - 3y^4}{6r_2(1-y^4)}. \end{aligned}$$

Similarly, from (A.8)

$$J(\beta) = \frac{2r_1}{3}r_2^2 y^3 (1-y)^2 \left(1 + y + y^2 + y^3\right).$$

Note that $J(\beta) \neq 0$ for each $y \in (0,1)$ (since $\beta < 0$, $y \in (0,1)$). Equation (16) can be expressed as

$$g(\beta) = 2v_2\delta + \frac{2a_2}{3} - \frac{1}{2r_2}\frac{4 - 3e^{\frac{r_2\beta}{2}}}{1 - e^{\frac{r_2\beta}{2}}} - \beta = 0. \tag{A.24}$$

[51] We are not assuming that β^* exists, but merely assuming what should be the sign of it in case it does exists.

From (A.24), the derivative of $g(\beta)$ is

$$g'(\beta) = -\frac{-7e^{\frac{r_2\beta}{2}} + 4e^{r_2\beta} + 4}{4\left(e^{\frac{r_2\beta}{2}} - 1\right)^2} = -\frac{4y^2 - 7y + 4}{4(1-y)^2} < 0 \text{ for all } \beta < 0.$$

Thus, g is a strictly decreasing function for $\beta \in (-\infty, 0)$. Moreover,

$$\lim_{\beta \to -\infty} g(\beta) = \infty \text{ and } \lim_{\beta \to 0^-} g(\beta) = -\infty.$$

It follows that there exists a unique $\beta^* < 0$ that satisfies equation $g(\beta^*) = 0$. From Theorem 1, the existence of the SE follows.

From (17), equilibrium prices are

$$\begin{aligned} p_1^* &= \frac{1}{r_1} - \frac{a_2}{2}, \\ p_2^* &= \frac{1}{r_2(1-y^4)} - \frac{1}{2}a_1, \text{ and} \\ p_{2m}^* &= \frac{6 + y + y^2 + y^3 - 3y^4}{6r_2(1-y^4)} - \frac{1}{6}a_1, \end{aligned} \tag{A.25}$$

and equilibrium market shares are

$$\begin{aligned} n_1^* &= \frac{1}{3}, \\ n_2^* &= \frac{1}{3}\left(1 - y^3(4-3y)\right), \text{ and} \\ n_2^{m*} &= y^3(4-3y). \end{aligned} \tag{A.26}$$

If $\beta \geq 0$, from (A.22) and (A.23), $n_2 = F^1(0, \beta) = 0$ and $n_2^m = F^2(-\beta, \beta) = 1$. In this case, $J(\beta) = 0$. Therefore, we cannot apply Theorem 1. Instead, we would have to solve the new platform's problem given by

$$\max_{\{p_1, p_2, p_{2m}\}} \pi^1(p_1, p_2, p_{2m}) = n_1 p_1 + p_{2m},$$

subject to $\beta = u_2^m - u_2 = 2v_2\delta + \frac{2a_2}{3} + p_2 - p_{2m} - 2p_{2m}^* \geq 0$. However, this problem has no finite solution in prices, since by letting $p_1 \to 0$, $p_2 \to \infty$ and $p_{2m} \to \infty$, $\pi^1(p_1, p_2, p_{2m}) \to \infty$.

(ii) To show that the second-order conditions are satisfied, we show that inequalities (a)-(c) in Lemma 1 hold true. Note that (a) follows from the fact that H_1, given by (A.21), is log-concave. From (A.21),

$$d_{11}=\begin{cases}-\frac{1}{n_1r_1} & n_1\leq 1/3\\ -\frac{4\left(\frac{1}{2}\left(3-\sqrt{3}\sqrt{4n_1-1}\right)^3-\frac{9}{4}\left(3-\sqrt{3}\sqrt{4n_1-1}\right)^2-\frac{3}{2}\left(3-\sqrt{3}\sqrt{4n_1-1}\right)+9\right)}{\sqrt{3}(4n_1-1)^{3/2}\left(3-\sqrt{3}\sqrt{4n_1-1}\right)^2r_1} & n_1>1/3\end{cases}. \tag{A.27}$$

From (A.27), $\min_{n_1\in[0,1]}\{-d_{11}\}=\frac{3}{r_1}$. From the full market coverage assumption and (A.19), $d_{22}=2\left(p_2\right)_2+n_2\left(p_2\right)_{22}+n_2^m\left(p_{2m}\right)_{22}$ is a function only of n_2^m. From (A.22) and (A.23), it follows that $\min_{n_2^m\in[0,1]}\{-d_{22}\}=\frac{3}{r_2}$. Assumption 2 implies that

$$d_{11}d_{22}>\frac{9}{r_1r_2}>\frac{9}{4}\left(a_1+a_2\right)^2.$$

Thus, (b) in Lemma 1 is satisfied. Similarly, $(d_{22}d_{33}-d_{23}^2)$ and $\left(d_{33}-\frac{2}{3}d_{23}+\frac{1}{9}d_{22}\right)$ are functions only of n_2^m. From (A.22) and (A.23), it follows that

$$\min_{n_2^m\in[0,0.93]}\left\{-\frac{d_{22}d_{33}-d_{23}^2}{d_{33}-\frac{2}{n}d_{23}+\frac{1}{n^2}d_{22}}\right\}=\frac{1}{r_2} \tag{A.28}$$

Assumption 2 implies that

$$d_{11}\left(\frac{d_{22}d_{33}-d_{23}^2}{d_{33}-\frac{2}{n}d_{23}+\frac{1}{n^2}d_{22}}\right)>\frac{3}{r_1r_2}>\frac{9}{4}\left(a_1+a_2\right)^2.$$

Since $\left(d_{33}-\frac{2}{3}d_{23}+\frac{1}{9}d_{22}\right)<0$ for all n_2^m, it follows that (c) in Lemma 1 holds true. Thus, the second-order conditions are satisfied for any $n_1\in[0,1]$, and $n_2^m\in[0,0.93]$.

Proof of Corollary 1. From (A.25) and the fact that $y\in(0,1)$, it follows that

$$\begin{aligned}p_2^*-3p_{2m}^*&=-\frac{4-3y}{2r_2\left(1-y\right)}<0,\\ p_2^*-p_{2m}^*&=-\left(\frac{a_1}{3}+\frac{y\left(3y^2+2y+1\right)}{6r_2\left(y^3+y^2+y+1\right)}\right)<0.\end{aligned}$$

(i) If we differentiate with respect to r_2 in both sides of (A.24), by the Implicit Function Theorem

$$\frac{\partial\beta^*}{\partial r_2}=\frac{1}{r_2^2}\left(\frac{1}{y+4\left(1-y\right)^2}\right)\left(\underbrace{2\left(4-3y\right)\left(1-y\right)-r_2y\beta^*}_{>0,\text{ since }y\in(0,1)}\right)>0. \tag{A.29}$$

(ii) First we show that $\frac{\partial y}{\partial r_2} > 0$. Recall that $\frac{\partial y}{\partial r_2} = \frac{1}{2} y \left(\beta^* + r_2 \frac{\partial \beta^*}{\partial r_2} \right)$. From (A.29),

$$\frac{\partial y}{\partial r_2} = \frac{y(1-y)(4-3y+2r_2(1-y)\beta^*)}{r_2 \underbrace{\left(4-7y+4y^2\right)}_{>0}}. \tag{A.30}$$

Note that (A.30) is positive if $(4-3y)+2r_2\beta^*(1-y) > 0$. Recall that β^* is implicitly defined by

$$2v_2\delta + \frac{2a_2}{3} - \frac{1}{2r_2}\frac{4-3y}{1-y} - \beta^* = 0,$$

which implies that

$$\frac{4-3y}{1-y} + 2r_2\beta^* = 2r_2\left(2v_2\delta + \frac{2a_2}{3}\right) > 0.$$

Thus, $\frac{\partial y}{\partial r_2} > 0$. From (A.26),

$$\frac{\partial n_2^{m*}}{\partial r_2} = \underbrace{\frac{\partial n_2^{m*}}{\partial y}}_{>0} \cdot \frac{\partial y}{\partial r_2} = 12(1-y)y^2 \cdot \frac{\partial y}{\partial r_2} > 0,$$

which concludes the proof.

Proof of Corollary 2. It follows directly from Proposition 3.

Proof of Corollary 3. Note that

$$p_2^* - p_{2,E}^* = \frac{1}{r_2\left(1-y^4\right)} - \frac{1}{r_2} = \frac{1}{r_2} \cdot \frac{y^4}{1-y^4} > 0. \tag{A.31}$$

(a) Consumer surplus is computed by comparing the utility, u_1^*, that consumers obtain in the exclusive–non-exclusive game versus the utility, $u_{1,E}^*$, they obtain in the exclusive-only game,

$$\begin{aligned} \Delta CS_1 &= (u_1^* - u_{1,E}^*) \\ &= \frac{2}{3}a_1\left(4y^3 - 3y^4\right) > 0, \qquad \text{since } y \in (0,1). \end{aligned} \tag{A.32}$$

Note that content provider surplus can be broken down into exclusive and non-exclusive groups. Exclusive content provider surplus is

$$\begin{aligned} \Delta CS_2^E &= (u_2^* - u_{2,E}^*) \\ &= -\frac{1}{r_2}\frac{y^4}{(1-y^4)} < 0, \qquad \text{since } y \in (0,1). \end{aligned} \tag{A.33}$$

Non-exclusive content provider surplus is

$$\begin{aligned}\Delta CS_2^{NE} &= (u_2^{m*} - u_{2,E}^*) \\ &= 2v_2\delta + \frac{2}{3}a_2 - 3p_{2m}^* + p_{2,E}^* + \mathbb{E}\left[\varepsilon_2^j + \varepsilon_2^l \middle| u_2^{m*} > \max_i u_2^*\right].\end{aligned} \tag{A.34}$$

To compute the conditional expected value $\mathbb{E}\left[\varepsilon_2^j + \varepsilon_2^l \middle| u_2^{m*} > \max_i u_2^*\right]$, we use the fact that in the symmetric equilibrium, $\max_i u_2^* = u_2^*$. It follows that

$$\begin{aligned}\mathbb{E}\left[\varepsilon_2^j + \varepsilon_2^l \middle| u_2^{m*} > u_2^*\right] &= \mathbb{E}\left[\varepsilon_2^j + \varepsilon_2^l \middle| 2v_2\delta + \frac{2}{3}a_2 + p_2^* - 3p_{2m}^* + \left(\varepsilon_2^j + \varepsilon_2^l\right) > 0\right] \\ &= \mathbb{E}\left[\varepsilon_2^j + \varepsilon_2^l \middle| \varepsilon_2^j + \varepsilon_2^l > \underbrace{-\left(2v_2\delta + \frac{2}{3}a_2 + p_{2,E}^* - 3p_{2m}^*\right)}_{\equiv c_0}\right] \\ &= \mathbb{E}\left[\varepsilon_2^j + \varepsilon_2^l \middle| \varepsilon_2^j + \varepsilon_2^l > c_0\right] = \frac{c_0^2 r_2^2 + 2c_0 r_2 + 2}{r_2\left(c_0 r_2 + 1\right)},\end{aligned} \tag{A.35}$$

Note that $c_0 = -\beta^* + (p_2^* - p_{2,E}^*)$. From (A.31), $p_2^* > p_{2,E}^*$, it follows that $c_0 > 0$.

From (A.34) and (A.35)

$$\begin{aligned}\Delta CS_2^{NE} &= \frac{c_0 r_2 + 2}{r_2\left(c_0 r_2 + 1\right)} \\ &= \frac{1}{r_2} \cdot \left(\frac{6\left(1 - y^4\right)}{3\left(-3y^4 + y^3 + y^2 + y + 6\right) - 4r_2\left(a_2 + 3\delta v_2\right)\left(1 - y^4\right)} + 1\right) > 0.\end{aligned} \tag{A.36}$$

Finally, the difference in profits between the exclusive–non-exclusive and exclusive-only games is

$$\begin{aligned}\Delta\pi &= (n_1^* p_1^* + n_2^* p_2^* + n_2^{m*} p_{2m}^*) - \left(n_{1,E}^* p_{1,E}^* + n_{2,E}^* p_{2,E}^*\right) \\ &= \frac{1}{3}\underbrace{\left(p_2^* - p_{2,E}^*\right)}_{>0} + \frac{1}{3}\underbrace{\left(3p_{2m}^* - p_2^*\right)}_{>0} n_2^{m*} > 0.\end{aligned} \tag{A.37}$$

From (A.32), (A.33), (A.36) and (A.37), it follows that the introduction of non-exclusive contracts increases consumer surplus and platform profits, increases the surplus of content providers that multi-home, but decreases the surplus of those that single-home.

(b) ΔCS denotes the sum of consumer and content provider surpluses, $\Delta CS \equiv \Delta CS_1 + \Delta CS_2^E + \Delta CS_2^{NE}$. We show that $\Delta CS \to -\infty$ as $v_0 \to \infty$, and $\Delta CS > 0$ as $v_0 \to 0$, which proves (b). From (A.32), (A.33) and (A.36),

$$\Delta CS = \frac{2}{3}a_1\left(4y^3 - 3y^4\right) - \frac{1}{r_2}\frac{y^4}{(1-y^4)} + \frac{1}{r_2}\cdot\left(\frac{6\left(1-y^4\right)}{3\left(-3y^4+y^3+y^2+y+6\right) - 4r_2\left(a_2+3\delta v_2\right)\left(1-y^4\right)} + 1\right). \tag{A.38}$$

Note that when $v_0 = 2\delta v_2 + \frac{2}{3}a_2 \to \infty$, from (A.24), $\beta^* \to 0^-$ and $y \to 1^-$. Similarly, from (A.24)

$$\lim_{v_0\to\infty} v_0\left(1-y\right) = \frac{1}{2r_2}.$$

From (A.38), it follows that $\Delta CS \to -\infty$ as $v_0 \to \infty$. Similarly, as $v_0 \to 0$, from (A.24) $y \to y_0$, where y_0 is the unique solution of

$$\frac{4-3y_0}{4(1-y_0)} + \ln(y_0) = 0. \tag{A.39}$$

From (A.39), $y_0 \approx 0.326$.

As $v_0 \to 0$, ΔCS can be rewritten as $\Delta CS = \frac{2}{3}a_1\left(4y_0^3 - 3y_0^4\right) + \frac{1}{r_2}M\left(y_0\right)$, where

$$M(y_0) \equiv \frac{8y_0^8 - 2y_0^7 - 2y_0^6 - 2y_0^5 - 19y_0^4 + y_0^3 + y_0^2 + y_0 + 8}{\left(1-y_0^4\right)\left(-3y_0^4 + y_0^3 + y_0^2 + y_0 + 6\right)}.$$

Since $M(y_0) > 0$, as $v_0 \to 0$, $\Delta CS > 0$.

(c) W denotes the overall welfare, $W = \Delta CS + \Delta\pi$. Note that

$$\lim_{v_0\to\infty} W = \frac{8a_1r_2 + 23}{12r_2} > 0.$$

Also, as $v_0 \to 0$

$$\lim_{v_0\to 0^+} W = \underbrace{\frac{y_0^3\left(y_0\left(9y_0^4 - 15y_0^3 + y_0^2 + y_0 - 6\right) + 16\right)}{6r_2\left(1-y_0^4\right)}}_{>0 \text{ for any } y_0\in(0,1)} + \frac{2}{3}a_1\left(4y_0^3 - 3y_0^4\right) + \frac{1}{r_2}M\left(y_0\right) > 0.$$

Thus, overall welfare increases when either $v_0 \to 0$ or $v_0 \to \infty$.

Proof of Proposition 4. The proof has two steps: (i) We show that a symmetric equilibrium (SE) exists; (ii) We show that the second-order conditions are satisfied.

(i) We show that a symmetric equilibrium (SE) exists. We show that there exists a unique solution β^* of the equation (16) and that $J(\beta^*) \neq 0$, where J is given by (A.8). From Theorem 1, the existence of the SE follows.

From (A.4), $X_k = \varepsilon_k^1 - \max\{\varepsilon_k^2, \varepsilon_k^3\}$ for $k \in \{1,2\}$, $Y = -\varepsilon_2^2 - \varepsilon_2^3$, and $Z = \boldsymbol{\varepsilon_2} \cdot \mathbf{1}_n - \max\{\varepsilon_2^2, \varepsilon_2^3\}$. From (A.5), the distributions H^1, F^1, and F^2 are

$$H_1(x_0) = \mathbb{P}(X_1 \le x_0) = \begin{cases} \frac{(2t_1 - x_0)(t_1+x_0)^2}{3t_1^3} & x_0 = 0 \vee (t_1 + x_0 > 0 \wedge x_0 < 0) \\ 1 & t_1 \le x_0 \end{cases}, \tag{A.40}$$

$$\begin{aligned} F^1(x_0, y_0) &= \mathbb{P}(X_2 > x_0, Y > y_0) \\ &= \begin{cases} \frac{y_0^2(2t_2 - 2x_0 + y_0)}{4t_2^3} & x_0 = 0 \wedge y_0 \in (-t_2, 0) \\ -\frac{6y_0^2(t_2-x_0)+12y_0(t_2-x_0)^2+4(t_2-x_0)^3+y_0^3}{12t_2^3} & x_0 = 0 \wedge y_0 \in (-2t_2, -t_2) \\ 0 & x_0 = 0 \wedge y_0 \in (0,\infty) \\ \frac{1}{3} & x_0 = 0 \wedge y_0 \in (-\infty, -2t_2) \end{cases}, \end{aligned} \tag{A.41}$$

and

$$\begin{aligned} F^2(z_0, y_0) &= \mathbb{P}(Z > z_0, Y < y_0) \\ &= \begin{cases} \frac{12t_2^3 - 6t_2\left(y_0^2 + 2z_0^2\right) + y_0^3 + 6y_0^2 z_0 + 4z_0^3}{12t_2^3} & z_0 + y_0 = 0 \wedge y_0 \in (-t_2, 0) \\ \frac{(2t_2 - z_0)^3}{4t_2^3} & z_0 + y_0 = 0 \wedge y_0 \in (-2t_2, -t_2) \\ 1 & z_0 + y_0 = 0 \wedge y_0 \in (0,\infty) \\ 0 & z_0 + y_0 = 0 \wedge y_0 \in (-\infty, -2t_2) \end{cases}. \end{aligned} \tag{A.42}$$

Suppose that if a solution β^* of equation (16) exists, it must be the case that $\beta^* \in (-t_2, 0)$ (we study the other cases at the end of the proof). Suppose that $\beta \in (-t_2, 0)$, from (A.42), $x(\beta) = F^2(-\beta, \beta) = 1 - \frac{3\beta^2(\beta+2t_2)}{4t_2^3}$. From (A.15), (A.41), and (A.42) combined, we obtain

$$\begin{aligned} M_2(\beta) &= -\frac{t_2^2}{2\beta}, \text{ and} \\ M_{2m}(\beta) &= \frac{2\beta^3 - 4t_2^3 - \beta t_2^2 + 4\beta^2 t_2}{6\beta^2 + 8\beta t_2}. \end{aligned}$$

Similarly, from (A.8)

$$J(\beta) = \frac{1}{t_1} \cdot \frac{\beta^2(3\beta + 4t_2)}{2t_2^5}.$$

Note that $J(\beta) \neq 0$ since $\beta \in (-t_2, 0)$. Equation (16) becomes

$$g(\beta) = \frac{2\left(\beta t_2\left(4a_2 - 15\beta + 12\delta v_2\right) + 3\beta^2\left(a_2 - 3\beta + 3\delta v_2\right) + 6t_2^3\right)}{3\beta\left(3\beta + 4t_2\right)} = 0. \tag{A.43}$$

From (A.43), g' is given by

$$g'(\beta) = t_2^2\left(\frac{1}{\left(3\beta + 4t_2\right)^2} - \frac{1}{\beta^2}\right) - 2 < 0 \quad \text{for all } \beta \in (-t_2, 0).$$

Thus, g is a strictly decreasing function for $\beta \in (-t_2, 0)$. Moreover,

$$\lim_{\beta \to (-t_2)^+} g(\beta) = \frac{2}{3}\left(a_2 + 3\delta v_2\right) > 0 \;\text{ and }\; \lim_{\beta \to 0^-} g(\beta) = -\infty.$$

Therefore, there exists a unique $\beta^* \in (-t_2, 0)$ that satisfies equation $g(\beta^*) = 0$. From Theorem 1, the existence of the SE follows.

From (17), equilibrium prices are

$$\begin{aligned} p_1^* &= \frac{t_1}{3} - \frac{a_2}{2}, \\ p_2^* &= -\frac{t_2^2}{2\beta^*} - \frac{1}{2}a_1, \text{ and} \\ p_{2m}^* &= \frac{2\beta^{*3} - 4t_2^3 - \beta^* t_2^2 + 4\beta^{*2} t_2}{6\beta^{*2} + 8\beta^* t_2} - \frac{1}{6}a_1, \end{aligned}$$

and equilibrium market shares are

$$\begin{aligned} n_1^* &= \frac{1}{3}, \\ n_2^* &= \frac{\beta^{*2}(\beta^* + 2t_2)}{4t_2^3}, \text{ and} \\ n_2^{m*} &= 1 - \frac{3\beta^{*2}(\beta^* + 2t_2)}{4t_2^3}. \end{aligned} \tag{A.44}$$

If $\beta \in (0, \infty)$, from (A.41) and (A.42), $n_2 = 0$, $n_2^m = 1$ and $J(\beta) = 0$. Thus, we cannot apply Theorem 1. Instead, we would have to solve the new platform's problem given by

$$\max_{\{p_1, p_2, p_{2m}\}} \pi^1\left(p_1, p_2, p_{2m}\right) = n_1 p_1 + p_{2m},$$

subject to $\beta = u_2^m - u_2 = 2v_2\delta + \frac{2a_2}{3} + p_2 - p_{2m} - 2p_{2m}^* > 0$. However, this problem has no finite solution in prices, since by letting $p_1 \to 0$, $p_2 \to \infty$ and $p_{2m} \to \infty$, $\pi^1\left(p_1, p_2, p_{2m}\right) \to \infty$. If $\beta \in (-\infty, -2t_2)$, it is not difficult to show that no finite solution in prices exists. Finally, if $\beta \in (-2t_2, -t_2)$, $J(\beta) \neq 0$ and the solution of (16) is

$\beta^* = \frac{a_2}{3} - t_2 + \delta v_2$, which do not satisfy the imposed restriction $\beta^* \in (-2t_2, -t_2)$.

(ii) To show that the second-order conditions are satisfied, we show that inequalities (a)-(c) in Lemma 1 hold true. Note that (a) follows from the fact that H_1, given by (A.40), is log-concave. From (A.40) and (A.19),

$$\min_{n_1 \in [0,1]} \{-d_{11}\} = \frac{4}{3} t_1. \tag{A.45}$$

From the full market coverage assumption and (A.19), $d_{22} = 2 (p_2)_2 + n_2 (p_2)_{22} + n_2^m (p_{2m})_{22}$ is a function only of n_2^m. From (A.41) and (A.42), it follows that

$$\min_{n_2^m \in [0.25, 0.8]} \{-d_{22}\} > 2t_2. \tag{A.46}$$

By assumption $\frac{32}{27} t_1 t_2 > (a_1 + a_2)^2$, which implies that for any $n_1 \in [0,1]$ and $n_2^m \in [0.25, 0.8]$,

$$d_{11} d_{22} > \frac{8}{3} t_1 t_2 > \frac{9}{4} (a_1 + a_2)^2 .$$

Thus, (b) in Lemma 1 is satisfied. Similarly, $(d_{22} d_{33} - d_{23}^2)$ and $\left(d_{33} - \frac{2}{3} d_{23} + \frac{1}{9} d_{22}\right)$ are functions of n_2^m. From (A.41) and (A.42), it follows that

$$\min_{n_2^m \in [0.25, 0.8]} \left\{ - \frac{d_{22} d_{33} - d_{23}^2}{d_{33} - \frac{2}{n} d_{23} + \frac{1}{n^2} d_{22}} \right\} > 2t_2. \tag{A.47}$$

By assumption $\frac{32}{27} t_1 t_2 > (a_1 + a_2)^2$, which implies that

$$d_{11} \left(\frac{d_{22} d_{33} - d_{23}^2}{d_{33} - \frac{2}{n} d_{23} + \frac{1}{n^2} d_{22}} \right) > \frac{8}{3} t_1 t_2 > \frac{9}{4} (a_1 + a_2)^2 .$$

Since $\left(d_{33} - \frac{2}{3} d_{23} + \frac{1}{9} d_{22}\right) < 0$ for all $n_2^m \in [0.25, 0.8]$, it follows that (c) in Lemma 1 holds true. Thus, the second-order conditions are satisfied for any $n_1 \in [0,1]$, and $n_2^m \in [0.25, 0.8]$.

Proof of Corollary 4. In the Appendix C, we show that prices in the two-platform exclusive–non-exclusive game (with exponential idiosyncratic preferences) are given by

$$p_{2,2}^* = \frac{1}{r_2} - a_1 \quad \text{and} \quad p_{2m,2}^* = \frac{1}{r_2} - \frac{1}{2} a_1,$$

where $p_{2,n}^*$ and $p_{2m,n}^*$ denote the price that single- and multi-homing content providers pay, respectively, with n platforms.

(i) From (18), $p^*_{1,3} - p^*_{1,2} = \frac{a_2}{2} > 0$,

$$\begin{aligned} p^*_{2,3} - p^*_{2,2} &= \left(\frac{1}{r_2\left(1-y^4\right)} - \frac{1}{2}a_1\right) - \left(\frac{1}{r_2} - a_1\right) \\ &= \frac{a_1}{2} + \frac{y^4}{r_2\left(1-y^4\right)} > 0, \text{ and} \\ p^*_{2m,3} - p^*_{2m,2} &= \left(\frac{6+y+y^2+y^3-3y^4}{6r_2\left(1-y^4\right)} - \frac{1}{6}a_1\right) - \left(\frac{1}{r_2} - \frac{1}{2}a_1\right) \\ &= \frac{a_1}{3} + \frac{3y^4+y^3+y^2+y}{6r_2\left(1-y^4\right)} > 0. \end{aligned}$$

Thus, consumer and content provider prices increase as n goes from two to three platforms.

(ii) Let π^*_n be the equilibrium profits with n platforms. We show that $\Delta\pi = \pi^*_3 - \pi^*_2 > 0$. Note that

$$\begin{aligned} \Delta\pi &= \pi^*_3 - \pi^*_2 \\ &= \left(n^*_{1,3}p^*_{1,3} + n^*_{2,3}p^*_{2,3} + n^{m*}_{2,3}p^*_{2m,3}\right) - \left(n^*_{1,2}p^*_{1,2} + n^*_{2,2}p^*_{2,2} + n^{m*}_{2,2}p^*_{2m,2}\right) \\ &= \frac{1}{3}\left(a_1+a_2\right) - \frac{1}{6r_1} - \frac{1}{6r_2}\left[\frac{3\left(1+x^*_2\right)\left(1-y^4\right)-2}{1-y^4} - \frac{y^3\left(4-3y\right)^2}{\left(1-y\right)}\right], \end{aligned}$$

where $x^*_2 = e^{a_2r_2+2\delta v_2r_2-2}$. It follows that $\Delta\pi > 0 \iff$

$$2\left(a_1+a_2\right) - \frac{1}{r_1} - \frac{1}{r_2} > h(v_0, r_2), \tag{A.48}$$

where h is defined as

$$h(v_0, r_2) \equiv \frac{1}{r_2}\left[\frac{3\left(1+x^*_2\right)\left(1-y^4\right)-2}{1-y^4} - \frac{y^3\left(4-3y\right)^2}{\left(1-y\right)} - 1\right].$$

Thus, if a_1 is such (A.48) holds, then $\Delta\pi > 0$.

(iii) Let ΔCS_2 be the difference in the surplus for the exclusive content providers when the number of platforms increases from two to three in the exclusive–non-exclusive game. We show that $\Delta CS^E_2 < 0$. From Proposition 3,

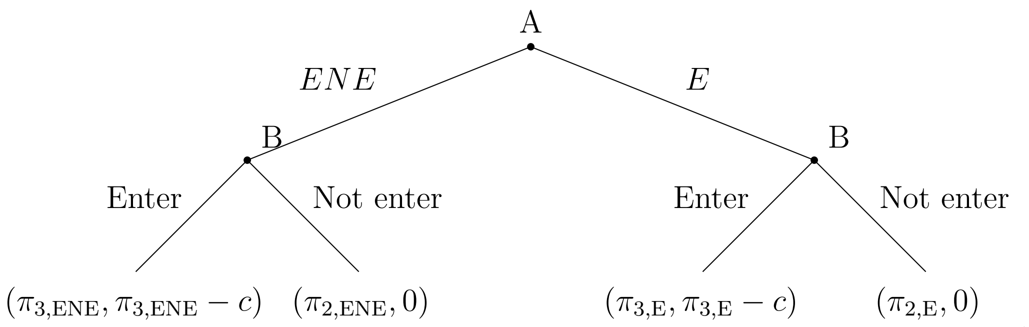
$$\begin{aligned} \Delta CS^E_2 &= \Delta u^*_2 \\ &= -\frac{1}{6}a_2 - \left(\frac{a_1}{2} + \frac{y^4}{r_2\left(1-y^4\right)}\right) < 0. \end{aligned} \tag{A.49}$$

(iv) Let ΔCS_2^m be the difference in the surplus for the non-exclusive content providers when the number of platforms increases from two to three in the exclusive–non-exclusive game. From Proposition 3,

$$\begin{aligned}\Delta CS_2^m &= (2v_2\delta - v_2\delta) - \left(3p^*_{2m,3} - 2p^*_{2m,2}\right) + \mathbb{E}\left[\varepsilon_2^i | u^{m*}_{2,3} > u^*_{2,3} \text{ and } u^{m*}_{2,2} > u^*_{2,2}\right] \\ &= v_2\delta - \frac{1}{2}a_1 - \underbrace{\frac{y\left(3y^3 + y^2 + y + 1\right)}{2r_2\left(1 - y^4\right)}}_{>0,\text{ since } y\in(0,1)} . \end{aligned} \tag{A.50}$$

From (A.50), if $a_1 > 2\delta v_2$, then $\Delta CS_2^m < 0$.

Proof of Proposition 5. The game described in Subsection 5.1 can be summarized by the following diagram:



Node A represents the decision of the two incumbent platforms and node B represents the decision of the entrant platform, where E refers to the exclusive-only pricing decision and ENE to the exclusive–non-exclusive pricing decision. The entry cost is c. It follows that:

(i) If $c \in (\pi_{3,\text{E}}, \pi_{3,\text{ENE}})$ and $\pi_{3,\text{ENE}} < \pi_{2,\text{E}}$, then the incumbent platforms offer exclusive-only contracts and the entrant does not enter;

(ii) if $c < \pi_{3,\text{E}}$ or $c > \pi_{3,\text{ENE}}$, the incumbent platforms offer exclusive–non-exclusive contracts and the entrant enters the market only if $c < \pi_{3,\text{E}}$.

Proof of Corollary 5. (i) Let $\Delta\pi_{\text{ENE}} \equiv \pi_{3,\text{ENE}} - \pi_{2,\text{ENE}}$ and $\Delta\pi_{\text{E}} \equiv \pi_{3,\text{E}} - \pi_{2,\text{E}}$ be the difference between platforms' profits when the number of platforms increases from two to three in the exclusive–non-exclusive and the exclusive-only games, respectively. From the proof of Corollary 4

$$\Delta\pi_{\text{ENE}} = \frac{1}{3}\left(a_1 + a_2\right) - \frac{1}{6r_1} - \frac{1}{6r_2}\left[\frac{3\left(1 + x_2^*\right)\left(1 - y^4\right) - 2}{1 - y^4} - \frac{y^3\left(4 - 3y\right)^2}{\left(1 - y\right)}\right], \tag{A.51}$$

and

$$\Delta\pi_{\mathrm{E}} = \frac{1}{6}\left(-\frac{1}{r_1} - \frac{1}{r_2} + 2a_2 + 2a_1\right). \tag{A.52}$$

From (A.51) and (A.52), it follows that

$$\begin{aligned} \Delta\pi_{\mathrm{E}} - \Delta\pi_{\mathrm{ENE}} &= \frac{1}{6r_2}\left[\frac{3\left(1+x_2^*\right)\left(1-y^4\right)-2}{1-y^4} - \frac{y^3\left(4-3y\right)^2}{\left(1-y\right)} - 1\right] \\ &= \frac{1}{6}h\left(v_0, r_2\right). \end{aligned} \tag{A.53}$$

From (A.53), if the number of platforms, n, increases from two to three, platforms are better off in the exclusive–non-exclusive game than in the exclusive-only game if and only if $h < 0$.

(ii) Let $\Delta CS^{E}_{2,\mathrm{ENE}}$ and $\Delta CS^{\mathrm{E}}_{2.E}$ be the content provider surpluses when the number of platforms increases from two to three in the exclusive–non-exclusive and the exclusive-only games, respectively. From the proof of Corollary 4

$$\begin{aligned} \Delta CS^{E}_{2,\mathrm{ENE}} - \Delta CS^{\mathrm{E}}_{2.E} &= -\frac{1}{6}a_2 - \left(\frac{a_1}{2} + \frac{y^4}{r_2\left(1-y^4\right)}\right) - \left(-\frac{a_2}{6} - \frac{a_1}{2}\right) \\ &= -\frac{y^4}{r_2\left(1-y^4\right)} < 0. \end{aligned} \tag{A.54}$$

Thus, exclusive content providers are worse off in the exclusive–non-exclusive game than in the exclusive-only game.

B Tables and Figures

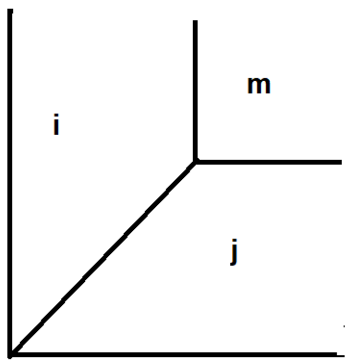
Table B1: Video Streaming Exclusive and Non-Exclusive Titles

Amazon Exclusive	Hulu Exclusive	Netflix Exclusive	Non-Exclusive
The Grand Tour	Handmaid's Tale	Narcos	Disney content
Mozart in the Jungle	11.22.63	House of Cards	Mr. Robot
The Tick	Mindy Project	Stranger Things	One Punch
The Wire	Futurama	Black Mirror	Friends
Lore	Always Sunny	Santa Clarita Diet	Movies

Table B2: Video Games Exclusive and Non-Exclusive Titles

PlayStation Exclusive	Nintendo Exclusive	Xbox Exclusive	Non-Exclusive
Little Big Planet	Mario series	Kinect series	Final Fantasy
Crash Bandicoot	Zelda series	Halo	Guitar Hero
Kingdom Hearts	Wii Remote series	Gears of War	Just Dance
Grand Tourismo	Splatoon	Forza Horizon	Portal
Metal Gear Solid	Harvest Moon	Cuphead	Call of Duty

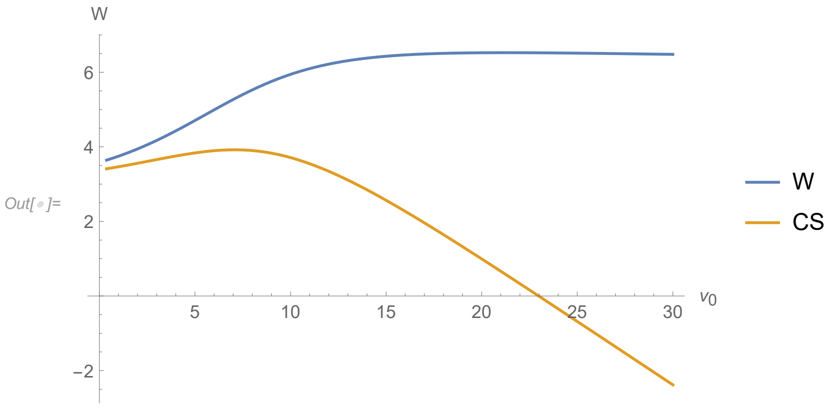
Figure B1: Two-Platform Exclusive–Non-Exclusive Market Share



Note: Users with high ϵ_i will exclusively join platform i. Users with high ϵ_j will exclusively join platform j; whereas users with high ϵ_i and ϵ_j will multi-home and join both platforms.

Under the assumptions of the exponential distribution game with three platforms. After the introduction of non-exclusive contracts, Figure B2 shows the sum of the consumer and content provider surpluses (orange line) and the overall welfare (blue line); i.e., the sum of the consumer and content provider surpluses plus platform profit surplus as functions of v_0. The set of parameters is given by $r_1 = 0.4$, $r_2 = 0.4$, and $a_1 = 2$, while $v_0 \equiv \frac{2}{3}a_2 + 2\delta v_2 \in (0, 30)$.

Figure B2: Sum of the Consumer and Content Provider Surpluses and the Overall Welfare



Note: Figure B2 shows the sum of the consumer and content provider surpluses (orange line) and the overall welfare (blue line) as functions of v_0. For this graph, we have fixed values of $r_1 = 0.4$, $r_2 = 0.4$, and $a_1 = 2$.

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ONLINE APPENDIX (Not for Publication)

C Two Platforms: Exponential Distribution

Similar to Subsection 4.1, we focus on linear network benefit functions— $\phi_1(n_2^i, n_2^m) = a_1(n_2^i + n_2^m)$, $\phi_2(n_1^i) = a_2 n_1^i$, and $\phi_m(n_1^1, n_1^2, n_1^3) = a_2(n_1^1 + n_1^2 + n_1^3)$, where a_1 and a_2 are positive constants. We assume that for each $i \in \{1,2\}$ and $k \in \{1,2\}$, ε_k^i follows an exponential distribution with parameter $r_k > 0$. For each side of the market, $k \in \{1,2\}$, ε_k^1 and ε_k^2 are i.i.d.

Proposition C.1. *Assume that* $\frac{1}{r_2} > \frac{1}{2}a_2 + \delta v_2$ *and* $\frac{1}{2r_1 r_2} > (a_1 + a_2)^2$. *There exists a unique* $\beta^* < 0$ *such that equation (16) holds and* $\beta^* = \frac{a_2}{2} - \frac{1}{r_2} + \delta v_2$. *Moreover, a unique symmetric subgame perfect Nash equilibrium exists in which, in stage 1, all platforms charge prices*

$$
\begin{aligned}
p_1^* &= \underbrace{\frac{1}{r_1}}_{\equiv M_1} - a_2, \\
p_2^* &= \underbrace{\frac{1}{r_2}}_{\equiv M_2} - a_1, \text{ and} \\
p_{2m}^* &= \underbrace{\frac{1}{r_2}}_{\equiv M_{2m}} - \frac{1}{2}a_1.
\end{aligned}
\tag{C.1}
$$

In stage 2, the market shares are $n_1^* = \frac{1}{2}$, $n_2^* = \frac{1}{2}\left(1 - e^{2r_2\beta^*}\right)$, *and* $n_2^{m*} = e^{2r_2\beta^*}$.

Note that from Proposition C.1, the market power terms M_2 and M_{2m} are equal, which implies that platforms exert the same market power on multi-homing and single-homing content providers. The subsidy term in p_2^*, a_1, is two times that of p_{2m}^*, $a_1/2$, so content providers receive the same subsidy regardless of whether they decide to single-home or multi-home. Since $\beta^* < 0$, in equilibrium, content providers obtain smaller deterministic utility by multi-homing rather than single-homing. However, multi-homing content providers are compensated by the realizations of positive idiosyncratic preferences. We now compare changes in prices and social surpluses upon the introduction of non-exclusive contracts in the following corollary.

Corollary C.1. *The introduction of non-exclusive contracts: (a) increases platform profits, consumer surplus, and content provider surplus of those who multi-home; (b) does* *not affect content provider surplus of those who single-home; (c) increases the overall welfare.*

Corollary C.1 shows that platforms, consumers, and multi-homing content providers gain from introduction of non-exclusive contracts. Consumers and single-homing content providers pay the same price as in the exclusive-only game, but consumers gain access to the new content provided by multi-homers. Single-homing content providers have access to the same amount consumers on side 1. Platforms' profits increase because non-exclusive content providers are being charged more than exclusive content providers.

Even though multi-homing content providers pay a higher price ($p_2^* < p_{2m}^*$), they receive multiple realizations of positive idiosyncratic preferences. The gain from these positive realizations of idiosyncratic preferences outweighs the exclusive premium payment ($p_{2m}^* - p_2^* > 0$), so multi-homing content providers are better off than before.

Since platform profits increase, consumer and multi-homing content provider surpluses increase, and single-homing content provider surplus remains constant. Thus, from Corollary C.1, it follows that the introduction of non-exclusive contracts is a welfare enhancing tool.

Proofs of Section C

Proof of Proposition C.1. The proof has two steps: (i) We show that a symmetric equilibrium (SE) exists; (ii) We show that the second-order conditions are satisfied.

(i) We show that there exists a unique solution β^* of the equation (16) and that $J(\beta^*) \neq 0$ (defined by A.8). Then, from Theorem 1, the existence of the SE follows.

From (A.4), $X_k = \varepsilon_k^1 - \varepsilon_k^2$ for $k \in \{1, 2\}$, $Y = -\varepsilon_2^2$, and $Z = \varepsilon_2^1$. From (A.5), the distributions H^1, F^1, and F^2 are

$$H_1(x) = \mathbb{P}(X_1 \leq x_0) = \begin{cases} \frac{e^{r_1 x_0}}{2} & x_0 \leq 0 \\ 1 - \frac{e^{-r_1 x_0}}{2} & x_0 > 0 \end{cases}, \tag{C.2}$$

$$F^1(x_0, y_0) = \mathbb{P}(X_2 > x_0, Y > y_0) = \begin{cases} \frac{1}{2} e^{-r_2 x_0} \left(1 - e^{2 r_2 y_0}\right) & x_0 \geq 0 \wedge y_0 < 0 \\ 0 & y_0 \geq 0 \end{cases}, \tag{C.3}$$

and

$$F^2(z_0, y_0) = \mathbb{P}(Z > z_0, Y < y_0) = \begin{cases} e^{r_2 (y_0 - z_0)} & y_0 \leq 0 \wedge z_0 \geq 0 \\ 1 & y_0 > 0 \wedge z_0 < 0 \end{cases}. \tag{C.4}$$

Suppose that any solution β of equation (16) must be negative.[52] From (A.5), $x(\beta) = F^2(-\beta, \beta) = e^{2r_2\beta}$. From (A.15), (C.3), and (C.4) we get that

$$M_2(\beta) = M_{2m}(\beta) = \frac{1}{r_2}.$$

Similarly, from (A.8)

$$J(\beta) = \frac{r_1}{2} \cdot r_2^2 e^{2\beta r_2}.$$

Note that $J(\beta) \neq 0$ for each $\beta \in (-\infty, 0)$. Equation (16) can be expressed as

$$g(\beta) = \frac{a_2}{2} - \beta - \frac{1}{r_2} + \delta v_2 = 0. \tag{C.5}$$

From (C.5), $\beta^* = \frac{a_2}{2} - \frac{1}{r_2} + \delta v_2$. Since $\frac{a_2}{2} + \delta v_2 < \frac{1}{r_2}$, it follows that $\beta^* < 0$. From Theorem 1, the existence of the SE follows. From (17), equilibrium prices are

$$\begin{aligned} p_1^* &= \frac{1}{r_1} - a_2, \\ p_2^*(\beta^*) &= \frac{1}{r_2} - a_1, \text{ and} \\ p_{2m}^*(\beta^*) &= \frac{1}{r_2} - \frac{1}{2}a_1, \end{aligned} \tag{C.6}$$

and equilibrium market shares are $n_1^* = \frac{1}{2}$, $n_2^* = \frac{1}{2}\left(1 - e^{2r_2\beta^*}\right)$, and $n_2^{m*} = e^{2r_2\beta^*}$.

(ii) To show that the second-order conditions are satisfied, we show that inequalities (a)-(c) in (A.20) hold true. Note that (a) follows from the fact that H_1, given by (C.2), is log-concave. From (C.2), (C.3), and (C.4), if $n_1 < 1/2$, then (b) and (c) are equivalent to

$$\frac{1}{2r_1r_2n_1(2n_2 + n_2^m)} > (a_1 + a_2)^2. \tag{C.7}$$

If $n_1 \geq 1/2$, (b) and (c) are equivalent to

$$\frac{2 - n_1}{2(n_1 - 1)^2 r_1r_2(2n_2 + n_2^m)} > (a_1 + a_2)^2. \tag{C.8}$$

If $\frac{1}{2r_1r_2} > (a_1 + a_2)^2$ then (C.7) and (C.8) hold true. Thus, (a)-(c) hold true which and the second-order conditions are satisfied.

[52] Similar to the proof of Proposition 3, assuming $\beta \geq 0$ would imply that no finite solution in prices exists.

Proof of Corollary C.1. Note that

$$p_{1,E}^* = \frac{1}{r_1} - a_2 \quad \text{and} \quad p_{2,E}^* = \frac{1}{r_2} - a_1.$$

Consumer surplus is computed by comparing the utility, u_1^*, that consumers obtain in the exclusive–non-exclusive game versus the utility, $u_{1,E}^*$, they obtain in the exclusive-only game,

$$\begin{aligned} \Delta CS_1 &= (u_1^* - u_{1,E}^*) \\ &= a_1 \cdot \frac{1}{2} e^{2r_2\beta^*} > 0. \end{aligned} \tag{C.9}$$

Note that content provider surplus can be broken down into exclusive and non-exclusive content groups. Exclusive content provider surplus is

$$\Delta CS_2^E = (u_2^* - u_{2,E}^*) = 0. \tag{C.10}$$

Non-exclusive content provider surplus is

$$\begin{aligned} \Delta CS_2^{NE} &= (u_2^{m*} - u_{2,E}^*) \\ &= v_2\delta + \frac{1}{2}a_2 - 2p_{2m}^* + p_{2,E}^* + \mathbb{E}\left[\varepsilon_2^j | u_2^{m*} > \max_i u_2^*\right]. \end{aligned} \tag{C.11}$$

To compute the conditional expected value $\mathbb{E}\left[\varepsilon_2^j | u_2^{m*} > \max_i u_2^*\right]$, we use the fact that in the symmetric equilibrium $\max_i u_2^* = u_2^*$. It follows that

$$\begin{aligned} \mathbb{E}\left[\varepsilon_2^j | u_2^{m*} > \max_i u_2^*\right] &= \mathbb{E}\left[\varepsilon_2^j | \varepsilon_2^j > \underbrace{-\left(v_2\delta + \frac{1}{2}a_2 - \frac{1}{r_2}\right)}_{\equiv c_0}\right] \\ &= \frac{c_0 r_2 + 1}{r_2}, \end{aligned} \tag{C.12}$$

Note that $c_0 > 0$, since $\frac{a_2}{2} + \delta v_2 < \frac{1}{r_2}$. From (C.11) and (C.12)

$$\Delta CS_2^{NE} = \frac{1}{r_2} > 0. \tag{C.13}$$

Finally, the difference in profits between the exclusive–non-exclusive and exclusive-only games is

$$\begin{aligned}\Delta\pi &= (n_1^* p_1^* + n_2^* p_2^* + n_2^{m*} p_{2m}^*) - \left(n_{1,E}^* p_{1,E}^* + n_{2,E}^* p_{2,E}^*\right) \\ &= \frac{1}{2} \underbrace{\left(p_2^* - p_{2,E}^*\right)}_{=0} + \frac{1}{2} \underbrace{\left(2p_{2m}^* - p_2^*\right)}_{>0} n_2^{m*} \\ &= \frac{1}{2r_2} n_2^{m*}.\end{aligned} \tag{C.14}$$

Note that (a) follows from (C.9), (C.13), and (C.14). Item (b) follows from (C.10). Item (c) follows directly from (a) and (b).

D Uniform Distribution: Two and Three Platforms

In this section, we study another example in which equation (16) has a unique solution, and therefore satisfy the condition for which Theorem 1 guarantees the existence of a subgame perfect Nash equilibrium. We focus on linear network benefit functions.

D.1 Two-platform game

We assume that for each $i \in \{1,2\}$ and $k \in \{1,2\}$, ε_k^i follows an uniform distribution with parameter $t_k > 0$; that is, $\varepsilon_k^i \sim \mathrm{U}[0, t_k]$. For each side of the market, $k \in \{1,2\}$, the idiosyncratic preferences $\{\varepsilon_k^1, \varepsilon_k^2\}$ are i.i.d.

Proposition D.1. *Assume that* $t_2 > \frac{1}{2}a_2 + \delta v_2$ *and* $\frac{3}{20}t_1 t_2 > (a_1 + a_2)^2$*. There exists a unique* $\beta^* < 0$ *such that equation (16) holds,* $\beta^* = \frac{1}{4}(2\delta v_2 + a_2 - 2t_2)$*. Moreover, a unique symmetric subgame perfect Nash equilibrium exists in which, in stage 1, all platforms charge prices*

$$\begin{aligned} p_1^* &= \underbrace{\frac{t_1}{2}}_{\equiv M_1(0)} - a_2, \\ p_2^* &= \underbrace{\frac{t_2^2}{t_2 - \beta^*}}_{\equiv M_2(\beta^*)} - a_1, \text{ and} \\ p_{2m}^* &= \underbrace{\frac{2t_2^2 - \beta^{*2}}{2t_2 - 2\beta^*}}_{\equiv M_{2m}(\beta^*)} - \frac{1}{2}a_1. \end{aligned} \tag{D.1}$$

In stage 2, the market shares are $n_1^* = \frac{1}{3}$, $n_2^* = \frac{1}{2}(1-x)$*, and* $n_2^{m*} = x$*, where* $x = \frac{(\beta^* + t_2)^2}{t_2^2}$*.*

Note that from Proposition D.1, the market power term M_2 is larger than M_{2m}, which implies that platforms exert higher market power on single-homing content providers. The subsidy term in p_2^*, a_1, is two times that of p_{2m}^*, $a_1/2$, so content providers receive the same subsidy regardless of whether they decide to single-home or multi-home. In equilibrium, content providers obtain smaller deterministic utility by multi-homing rather than single-homing. However, multi-homing content providers are compensated by the positive realizations of $\{\varepsilon_2^1, \varepsilon_2^2\}$.

D.2 Three-platform game

Throughout this subsection, we will assume that for $i \in \{1,2,3\}$ and $k \in \{1,2\}$, ε_k^i follows a uniform distribution with parameter $t_k > 0$; that is, $\varepsilon_k^i \sim \mathrm{U}[0, t_k]$. In Subsection 4.2, we showed that there exists a unique $\beta^* < 0$ such that equation (16) holds, and that there is a unique symmetric subgame perfect Nash equilibrium of the exclusive–non-exclusive game (see Proposition 4). Next, we have a corollary for comparative statics.

Corollary D.1. *In an exclusive–non-exclusive equilibrium:*

(i) $\frac{\partial \beta^*}{\partial t_2} < 0$;

(ii) $\frac{\partial n_2^{m*}}{\partial t_2} < 0$.

Corollary D.1(i) shows that, as platforms become more differentiated (i.e, as t_2 increases), β^* decreases. Since content providers have stronger preferences for platforms, the attractiveness of the single-homing option increases. Therefore, the proportion of multi-homing content providers decreases (which is Part (ii) of the corollary).

We now compare changes in prices and social surpluses upon the introduction of non-exclusive contracts in the following corollary.

Corollary D.2. *The introduction of non-exclusive contracts:*

(a) *increases exclusive content provider prices;*

(b) *increases platform profits and consumer surplus. Decreases content provider surplus of the single-homers. Let $c_0 \equiv -\beta^* + (p_2^* - p_{2,E}^*)$ and note that $c_0 > 0$. If $c_0 - 2t_2 < 0$ (≥ 0); then, muli-homing content providers surplus increases (decreases);*

(c) *increases welfare for a_1 high enough.*[53]

Corollary D.2 shows that the introduction of non-exclusive contracts softens competition for content providers, since $p_2^* > p_{2,E}^*$. Consumers pay the same price as in the exclusive-only game, but gain access to the new content provided by multi-homers. Platforms' profits increase by two means: the higher price being charged to exclusive content providers; and the gains from charging more to non-exclusive content providers than to exclusive content providers. Exclusive content providers surplus decreases and multi-homing content providers surplus increases (decreases) if $c_0 - 2t_2 < 0$ (≥ 0). Finally, if the marginal network benefit a_1 is high, the gains in consumer surplus and platform profits are enough to outweigh the loss in content provider surplus, so that the overall welfare increases. Thus, non-exclusive contracts increase welfare

In our final result, we compare prices of the exclusive–non-exclusive game when the number of platforms increases from two to three.

[53]By a_1 high enough, we mean that there exists $\overline{a}_1 > 0$ such that for any $a_1 > \overline{a}_1$, welfare increases.

Corollary D.3. *Assume that $t_2 > \frac{1}{2}a_2 + \delta v_2$. In the exclusive–non-exclusive game, if n goes from two to three platforms and a_1 is high enough, then consumer and content provider prices increase.*[54]

Proofs Section D

Proof of Proposition D.1. The proof has two steps: (i) We show that a symmetric equilibrium (SE) exists; (ii) We show that the second-order conditions are satisfied.

(i) We show that there exists a unique solution β^* of equation (16) and that $J(\beta^*) \neq 0$ (defined by A.8). Then, from Theorem 1, the existence of the SE follows.

From (A.4), $X_k = \varepsilon_k^1 - \varepsilon_k^2$ for $k \in \{1, 2\}$, $Y = -\varepsilon_2^2$, and $Z = \varepsilon_2^1$. From (A.5), the distributions H^1, F^1, and F^2 are

$$H_1(x) = \mathbb{P}(X_1 \leq x_0) = \begin{cases} \frac{t_1^2 + 2t_1 x_0 - x_0^2}{2t_1^2} & x_0 = 0 \vee (x_0 < t_1 \wedge x_0 > 0) \\ 1 & x_0 \geq t_1 \end{cases}, \tag{D.2}$$

$$F^1(x_0, y_0) = \begin{cases} -\frac{y_0(2t_2 - 2x_0 + y_0)}{2t_2^2} & x_0 = 0 \wedge y_0 \in (-t_2, 0) \\ \frac{1}{2} & x_0 = 0 \wedge y_0 \in (-\infty, -t_2) \\ 0 & x_0 = 0 \wedge y_0 \in (0, \infty) \end{cases}, \tag{D.3}$$

and

$$F^2(z_0, y_0) = \begin{cases} \frac{(t_2 + y_0)(t_2 - z_0)}{t_2^2} & z_0 \in (0, t_2) \wedge y_0 \in (-t_2, 0) \\ 0 & z_0 \in (t_2, \infty) \wedge y_0 \in (-\infty, -t_2) \\ 1 & z_0 \in (-\infty, 0) \wedge y_0 \in (0, \infty) \end{cases}. \tag{D.4}$$

Suppose that any solution β of equation (16) is such that $\beta \in (-t_2, 0)$. From (A.5), $x(\beta) = F^2(-\beta, \beta) = \frac{(\beta + t_2)^2}{t_2^2}$. From (A.15), (D.3), and (D.4) we get that

$$M_2(\beta) = \frac{t_2^2}{t_2 - \beta} \quad \text{and} \quad M_{2m}(\beta) = \frac{2t_2^2 - \beta^2}{2t_2 - 2\beta}.$$

Similarly, from (A.8)

[54] By a_1 high enough, we mean that there exists $\tilde{a}_1 > 0$ such that for any $a_1 > \tilde{a}_1$, content provider prices increase.

$$J(\beta) = \frac{1}{t_1} \cdot \frac{(t_2 - \beta)(\beta + t_2)}{t_2^4}.$$

Note that $J(\beta) \neq 0$ for each $\beta \in (-t_2, 0)$. Equation (16) can be expressed as

$$g(\beta) = \frac{a_2}{2} - 2\beta - t_2 + \delta v_2 = 0. \tag{D.5}$$

From (D.5), $\beta^* = \frac{1}{4}(2\delta v_2 + a_2 - 2t_2)$. Since $\delta v_2 + \frac{1}{2}a_2 < t_2$, it follows that $\beta^* \in (-t_2, 0)$. From Theorem 1, the existence of the SE follows.

From (17), equilibrium prices are

and equilibrium market shares are $n_1^* = \frac{1}{3}$, $n_2^* = \frac{1}{2}(1 - x)$, and $n_2^{m*} = x$, where $x = \frac{(\beta^* + t_2)^2}{t_2^2}$.

(ii) To show that the second-order conditions are satisfied, we show that inequalities (a)-(c) in (A.20) hold true. Note that (a) follows from the fact that H_1, given by (D.2), is log-concave. From (D.2), (D.3), and (D.4), if $n_1 < 1/2$, then (b) is equivalent to

$$\frac{1}{\sqrt{2}\sqrt{n_1}t_1} \frac{3\left(3n_2^m - 14\sqrt{n_2^m} + 14\right)t_1^2 t_2}{2\left(2 - \sqrt{n_2^m}\right)^3} > 4\left(a_1 + a_2\right)^2, \tag{D.6}$$

and (c) is equivalent to

$$\frac{60\left(n_2^m\right)^{\frac{3}{2}} - 9\left(n_2^m\right)^2 - 126n_2^m + 84\sqrt{n_2^m} - 1}{8\sqrt{2}\sqrt{n_1}\left(\sqrt{n_2^m} - 2\right)^4\left(n_2^m\right)^{\frac{1}{2}}} t_1 t_2 > \left(a_1 + a_2\right)^2. \tag{D.7}$$

It follows that for any $n_2^m > 0.04$, $\frac{3}{20}t_1 t_2 > (a_1 + a_2)^2$ implies that (D.6) and (D.7) hold true.

If $n_1 \geq 1/2$, (b) is equivalent to

$$\frac{4 - 3n_1}{\sqrt{2}\sqrt{1 - n_1}\left(1 - n_1\right)} \cdot \frac{3n_2^m - 14\sqrt{n_2^m} + 14}{2\left(2 - \sqrt{n_2^m}\right)^3} t_1 t_2 > 4\left(a_1 + a_2\right)^2, \tag{D.8}$$

and (c) is equivalent to

$$\frac{\left(4 - 3n_1\right)\left(60\left(n_2^m\right)^{3/2} - 9\left(n_2^m\right)^2 - 126n_2^m + 84\sqrt{n_2^m} - 1\right)}{24\sqrt{2}\left(1 - n_1\right)^{3/2}\left(\sqrt{n_2^m} - 2\right)^4\sqrt{n_2^m}} t_1 t_2 > \left(a_1 + a_2\right)^2. \tag{D.9}$$

It follows that for any $n_2^m > 0.04$, $\frac{3}{20}t_1 t_2 > (a_1 + a_2)^2$ implies that (D.8) and (D.9) hold true.

In conclusion, if $\frac{3}{20}t_1t_2 > (a_1+a_2)^2$, then the second-order conditions are satisfied for any $n_2^m > 0.04$.

Proof of Corollary D.1. (i) From the Implicit Function Theorem and Proposition 4,

$$\frac{\partial \beta^*}{\partial t_2} = \frac{\beta^*\left(-3\beta^{*3}+16t_2^3+18\beta^* t_2^2\right)}{9\beta^{*4}+8t_2^4+12\beta^* t_2^3+20\beta^{*2}t_2^2+24\beta^{*3}t_2} < 0, \tag{D.10}$$

since $\beta^* \in (-t_2, 0)$.

(ii) Note that

$$\frac{\partial n_2^{m*}}{\partial t_2} = \frac{3\beta^*\left(3\beta^*+4t_2\right)\left(\beta^* - t_2\frac{\partial \beta^*}{\partial t_2}\right)}{4t_2^4} < 0.$$

The latter inequality holds true if and only if $\frac{\beta^*}{t_2} > \frac{\partial \beta^*}{\partial t_2}$, which is true by (D.10) and the fact that $\beta^* \in (-t_2, 0)$.

Proof of Corollary D.2. (a) Let p_2^* be the exclusive price of the exclusive–non-exclusive game, and $p_{2,E}^*$ be the content provider price in the exclusive-only game. By Proposition 4,

$$p_2^* - p_{2,E}^* = -\frac{t_2^2}{2\beta^*} - \frac{t_2}{3} = -\underbrace{\frac{t_2(2\beta^*+3t_2)}{6\beta^*}}_{\text{Since } \beta^*\in(-t_2,0)} > 0.$$

(b) Consumer surplus is computed by comparing the utility, u_1^*, that consumers obtain in the exclusive–non-exclusive game versus the utility, $u_{1,E}^*$, they obtain in the exclusive-only game,

$$\begin{aligned}\Delta CS_1 &= (u_1^* - u_{1,E}^*)\\ &= \frac{1}{6}a_1\left(4 - \frac{3\beta^{*2}\left(\beta^*+2t_2\right)}{t_2^3}\right) > 0.\end{aligned} \tag{D.11}$$

Note that content provider surplus can be broken down into exclusive and non-exclusive groups. Exclusive content provider surplus is

$$\begin{aligned}\Delta CS_2^E &= (u_2^* - u_{2,E}^*)\\ &= \frac{t_2(2\beta^*+3t_2)}{6\beta^*} < 0.\end{aligned} \tag{D.12}$$

Non-exclusive content provider surplus is

$$\begin{aligned}\Delta CS_2^{NE} &= (u_2^{m*} - u_{2,E}^*) = \\ &= 2v_2\delta + \frac{2}{3}a_2 - 3p_{2m}^* + p_{2,E}^* + \mathbb{E}\left[\varepsilon_2^j + \varepsilon_2^l \middle| u_2^{m*} > \max_i u_2^*\right].\end{aligned} \tag{D.13}$$

To compute the conditional expected value $\mathbb{E}\left[\varepsilon_2^j + \varepsilon_2^l \middle| u_2^{m*} > \max_i u_2^*\right]$, we use the fact that in the symmetric equilibrium $\max_i u_2^* = u_2^*$. It follows that

$$\begin{aligned}\mathbb{E}\left[\varepsilon_2^j + \varepsilon_2^l \middle| u_2^{m*} > u_2^*\right] &= \mathbb{E}\left[\varepsilon_2^j + \varepsilon_2^l \middle| \varepsilon_2^j + \varepsilon_2^l > \underbrace{-\left(2v_2\delta + \frac{2}{3}a_2 + p_{2,E}^* - 3p_{2m}^*\right)}_{\equiv c_0}\right] \\ &= \mathbb{E}\left[\varepsilon_2^j + \varepsilon_2^l \middle| \varepsilon_2^j + \varepsilon_2^l > c_0\right] = \begin{cases}\frac{2}{3}\left(c_0 + t_2\right) & c_0 < 2t_2 \wedge c_0 \geq t_2 \\ \frac{2\left(c_0^3 - 3t_2^3\right)}{3\left(c_0^2 - 2t_2^2\right)} & c_0 < t_2 \\ 0 & c_0 \geq 2t_2\end{cases},\end{aligned} \tag{D.14}$$

Note that $c_0 = -\beta^* + \left(p_2^* - p_{2,E}^*\right) > 0$. From (D.13) and (D.14),

$$\Delta CS_2^{NE} = \begin{cases}\frac{1}{3}\left(2t_2 - c_0\right) & c_0 < 2t_2 \wedge c_0 \geq t_2 \\ -\frac{c_0^3 + 6t_2^2(t_2 - c_0)}{3\left(c_0^2 - 2t_2^2\right)} & c_0 < t_2 \\ -c_0 & c_0 \geq 2t_2\end{cases}. \tag{D.15}$$

Finally, the difference in profits between the exclusive–non-exclusive and exclusive-only games is

$$\begin{aligned}\Delta\pi &= (n_1^* p_1^* + n_2^* p_2^* + n_2^{m*} p_{2m}^*) - \left(n_{1,E}^* p_{1,E}^* + n_{2,E}^* p_{2,E}^*\right) \\ &= \frac{1}{3}\underbrace{\left(p_2^* - p_{2,E}^*\right)}_{>0,\text{ by Corollary D.2 (i)}} + \frac{1}{3}\underbrace{\left(3p_{2m}^* - p_2^*\right)}_{>0} n_2^{m*} > 0.\end{aligned} \tag{D.16}$$

From (D.11) and (D.16), it follows that consumer surplus and platform profits increase. From (D.12), content provider surplus of the single-homers decreases. From (D.15), multi-homing content provider surplus increases (decreases) if $c_0 - 2t_2 < 0$ (≥ 0).

(c) Welfare W is

$$W = \underbrace{\frac{1}{6}a_1\left(4 - \frac{3\beta^{*2}\left(\beta^* + 2t_2\right)}{t_2^3}\right)}_{>0} + \underbrace{\frac{t_2(2\beta^* + 3t_2)}{6\beta^*}}_{<0} + \begin{cases} \frac{1}{3}\left(2t_2 - c_0\right) & c_0 < 2t_2 \wedge c_0 \geq t_2 \\ -\frac{c_0^3 + 6t_2^2(t_2 - c_0)}{3\left(c_0^2 - 2t_2^2\right)} & c_0 < t_2 \\ -c_0 & c_0 \geq 2t_2 \end{cases}.$$

$$+ \underbrace{\Delta\pi}_{>0}$$

Note that c_0, β^*, and $\Delta\pi$ are independent of a_1. Thus, as $a_1 \to \infty$, $W \to \infty$. It follows that there exists $\bar{a}_1 > 0$ such that for any $a_1 > \bar{a}_1$, $W > 0$.

Proof of Corollary D.3. In Proposition D.1, we showed that prices in the two-platform exclusive–non-exclusive game (with uniform idiosyncratic preferences) are given by

$$p_{2,2}^* = \frac{t_2^2}{t_2 - \beta_2^*} - a_1 \quad \text{and} \quad p_{2m,2}^* = \frac{2t_2^2 - \beta_2^{*2}}{2t_2 - 2\beta_2^*} - \frac{1}{2}a_1,$$

where $p_{2,n}^*$ and $p_{2m,n}^*$ denote the price that single- and multi-homing content providers pay, respectively, with n platforms. From (20),

$$p_{2,3}^* - p_{2,2}^* = \frac{1}{2}\left(a_1 + t_2^2\left(\frac{8}{a_2 - 6t_2 + 2\delta v_2} - \frac{1}{\beta_3^*}\right)\right).$$

Note that $p_{2,3}^* - p_{2,2}^* \to \infty$ as $a_1 \to \infty$. Similarly, $p_{2m,3}^* - p_{2m,2}^* \to \infty$ as $a_1 \to \infty$, since

$$p_{2m,3}^* - p_{2m,2}^* = \frac{1}{18}\left(6a_1 + t_2^2\left(\frac{36}{a_2 - 6t_2 + 2\delta v_2} - \frac{9}{\beta_3^*} + \frac{2}{3\beta_3^* + 4t_2}\right) - \frac{9a_2}{4} + 6\beta_3^* - \frac{t_2}{2} - \frac{9\delta v_2}{2}\right).$$

It follows that there exists $\tilde{a}_1 > 0$ such that for any $a_1 > \tilde{a}_1$, content provider prices increase.