Dynamic Competition for Customer Memberships

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Abstract. A competitive two-period membership (subscription) market is analyzed. Two symmetric firms charge a "membership" fee that allows consumers to buy products or services at a given unit price for both periods. Firms can choose between long- or short-term memberships. When firms employ long-term membership, they have incentives to prevent their old customers from being poached by competitors, and to price-discriminate with their membership fee and unit price regarding customer purchase behavior. In contrast, with short-term membership, they do not discriminate between new and old customers with their unit price but only with their membership fees. Overall, the number of consumers poached is smaller with long-term memberships, but the equilibrium profits are higher when firms offer short-term memberships. Moreover, short-term membership is a Nash equilibrium.

Keywords: Competitive price discrimination, membership, dynamic competition.

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1 Introduction

In this paper, we study competition and consumer behavior in membership (subscription) markets. Generally, companies that implement a membership model charge a "membership" fee that allows consumers to buy products/services at a given unit price. A main feature of the membership markets is that transactions are not anonymous. Recent development of new information technologies has allowed firms to identify and classify consumers based on past purchase behavior and to price-discriminate according to this classification. Also, the structure of the tariffs and the strategies used vary for different markets and industries. In particular, memberships may be valid for multiple periods (long-term subscriptions) or for a short period of time (short-term subscriptions).[1] Firms may offer differentiated membership fees to their current and new customers (like *Amazon*, which offers a discounted membership fee to new customers) or may discriminate by offering differentiated unit prices or *usage* prices, like cable companies (e.g., *DirecTV* and *Spectrum*) or wireless carriers (e.g., *Sprint*) that offer cheaper monthly plans to new customers.[2]

Should firms choose long- or short-term memberships? When should firms discriminate with their membership fees or unit prices between new and current/old customers? How do different configurations of the tariff structure affect consumers' behavior and firms' ability to extract surplus? Surprisingly, there is little research investigating how firms select the best tariff in a competitive dynamic environment. Our paper provides a framework to investigate these questions.

As we explain in more detail below, we consider a competitive two-period model in which firm can choose between long- or short-term memberships, and forward-looking consumers buy from the firm that offers them the highest discounted utility. Our framework is based on the fact that more products and services are being offered to more people through memberships in new industries such as on-demand services (like grocery delivery services) and online marketplaces.[3] Under general assumptions, we show that in equilibrium, firms offer

[1]That is, for long-term memberships, consumers who buy from the same firm they bought from before do not need to pay the membership fee again, whereas for short-term memberships, consumers need to *renew* their memberships more frequently. For example, wireless carriers used to offer two-year plans while cable companies offered contracts for one or two years.

[2]*DirecTV* offers two-year contracts that guarantee a low monthly rate for the first year (for new customers), but does not commit to a unit price for the second year. When *Time Warner Cable* merged with *Spectrum*, the merged firm offered discounted monthly rates to customers who were not subscribed to applicable services within the previous 30 days.

[3]Membership (or subscription) business models are widely practiced in different industries. Previously dominated by the likes of credit card companies, telephone services, cable companies, and wireless carrier markets. Business-to-consumer subscription services have been growing at 200% annually since 2011 (McCarthy and Fader [16]), and have experienced an expansion from digital to physical goods—e.g., consumer packaged goods companies offer subscriptions for replenishments, and ridesharing services like *Uber* and *Lyft* offer subscriptions to their most active customers. Business-to-consumer subscription businesses sell a wide variety of products, from food (*Hello Fresh*, *Home Chef*, and *Blue Apron*) to grooming products (*Dollar Shave Club*, bought by *Unilever*) and clothes (*Le Tote* and *Rent the Runaway*). Moreover, delivery services have

short-term memberships. Intuitively, firms have more instruments to extract consumer surplus in period 2 when they offer short-term memberships, which allow them to distribute consumer surplus extraction more efficiently across the two periods. In contrast, with long-term memberships, firms are able to retain a larger share of their customers in period 2, but need to extract a larger portion of their profits in period 1.[4] Our framework also sheds light on when should firms discriminate with their membership fees or unit prices between new and old customers. We show that when firms offer long-term memberships, they discriminate with both their membership fee and unit prices. However, when firms offer short-term memberships, they discriminate between new and old customers only with their membership fees.[5]

So far, most of the literature has focused on models in which consumers have inelastic demand (i.e., they buy one unit), firms use linear pricing, and consumers are homogeneous in their taste for quality.[6] These three assumptions exclude from the analysis important features shared by most membership (subscription) markets described above. First, to understand the role of the membership fee and the unit price, we need to assume that consumers generally have elastic demands. Second, firms use richer tariff structures that are generally more complex than linear pricing.

For example, consider the online food ordering and delivery services. *Uber Eats*, *DoorDash*, and *Grubhub* offer subscription services that, for $9.99 per month, allow customers to buy from hundreds of restaurants with a $0 delivery fee on orders of $15 or more. *Uber Eats* and *DoorDash* provide a free trial (discount on the membership fee) for 30 days and Grubhub for 14 days. Similarly, two of the most important players in the grocery delivery services industry, *Instacart* and *Amazon Fresh*, used to charge a membership fee of $14.99 per month. The former provided a free trial (discount on the membership fee) for 14 days and the latter for 30 days, and both firms offered every-day low prices and free delivery on orders over $35

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expanded recently with *Instacart* and *Amazon*. This expansion of the membership business model into new industries also reveals the heterogeneity of the tariff structure employed across different industries.

[4]In some markets, firms predominantly use short-term memberships, like in the online food ordering and delivery services (e.g., *Uber Eats* and *Doordash*) and the online grocery delivery services (e.g., *Instacart*), while in other markets, long-term memberships are the norm, as in cable companies (e.g., *Spectrum* and *DirecTV*). There are at least three reasons that may affect our result—that under general assumptions, firms offer short-term memberships in equilibrium—and suggest why in reality we sometimes see long-term memberships. First, we assume that there is no switching cost, so the cost of transportation does not change if consumers go to a new or an old firm. Second, there could be behavioral reasons: once consumers sign up to a service, they become prey to the sunk cost fallacy and decide to keep using it. Finally, it may be costly for firms to offer multiple tariffs to old and new customers.

[5]Firms often discriminate with membership fees, like *Amazon* and *Instacart*, while there are other markets in which firms usually discriminate with their unit price, like in the cable market.

[6]The models proposed by Esteves and Reggiani [10] and Shin and Sudhir [20] are two exceptions. The first is a model in which consumers have constant elastic demands but only considers linear pricing, whereas the latter is a model in which the most valuable consumer buys more than one unit.

and \$40, respectively.[7] In the examples mentioned above, firms charge unit prices for their products (besides the membership fee) and consumers generally buy more than one unit of the products.

In other industries, memberships may be valid for multiple periods or even be life-time memberships. In these settings, firms may have incentives to discriminate between current and new customers not only with their membership fees, as in grocery delivery services, but also with their unit prices.[8] Cable companies and wireless carriers are good examples of these practices.[9] *DirecTV* offers two-year contracts that guarantee a low monthly rate for the first year (for new customers), but does not commit to a unit price for the second year.[10] *Spectrum*, which merged with *Time Warner Cable*, offered internet subscription services with lower monthly plans (or unit prices) to qualified customers who were not subscribed to applicable services within the previous 30 days, including old *Time Warner Cable* customers. Wireless carriers offer different monthly plan prices for new and old customers (e.g., *Sprint* offers lower monthly rates for new customers).[11]

In more detail, in this article, we analyze a competitive two-period membership model with symmetric firms offering horizontally differentiated brand products. Consumers are forward-looking and have variable demands. We study how the tariff structure (in terms of the length of the membership and discrimination between old and new customers with the membership fee or the unit price) charged by firms with no commitment affects prices, consumers welfare, firm profits, and the ability to poach customers from rivals.

We begin by introducing our benchmark model with long-term memberships. We start by considering a model in which consumers have private information about their horizontal brand preferences and *homogeneous* tastes for quality. In period 1, consumers decide to buy from either of the two firms and pay a membership fee and a unit price. In period 2, "old" customers don't need to pay the membership fee if they buy from the same firm they purchased from in period 1, but they pay a marginal price for the good and services they buy. However, if consumers decide to go to the rival firm, they have to pay a membership fee and a

[7] A FMI-Nielsen report predicts that online grocery spending could grow from 4.3% of total food and beverage sales spending in 2016, to as much as 20% percent share by 2025, which could reach upward of \$100 billion; as the report notes, "Put into context, that is the equivalent of nearly 3,900 grocery stores based on store volume" (Nielsen-FMI, 2017, "The Digitally Engaged Food Shopper").

[8] Note that in the online grocery industry, firms do not offer different unit prices to current and new customers.

[9] Credit card companies also offer introductory APR, annual fees, cash back, and miles, among others, usually conditional on spending a minimum amount in a given period of time.

[10] Specifically, *DirecTV* discloses that after 12 months or loss of eligibility, then "prevailing rate for TV package applies."

[11] The biggest carriers (e.g., *Verizon*, *T-Mobile*, and *AT&T*) have trade-in offers (subsidies in their membership fees), which include cash back and covering switching fees, among others, but do not discriminate with their unit price.

unit price.[12] Note that in period 2, the competition for old and new customers is asymmetric: each firm offers its old customers a unit price (linear pricing, LP) whereas the rival firm offers them a unit price and a membership fee (two-part tariff, 2PT). In equilibrium, in period 2, firms charge higher unit prices to their old customers and marginal-cost-based membership fees to their new customers. In period 1, firms charge marginal-cost-based membership fees. In general, membership fees allow firms to extract surplus more efficiently from consumers and prevent their old customers from being poached, making consumers worse off.[13]

Next, we present a short-term membership model. Here, consumers must renew the membership that allows them to buy product and services in the second period even if they go to the same firm they chose in period 1. We show that in equilibrium, firms offer marginal-cost pricing in both periods and extract surplus through membership fees paid by both new and old customers. Therefore, firms do not discriminate with their unit price between old and new customers, but they do so by offering differentiated membership fees.[14] We show that, in general, firms obtain higher profits by offering short-term rather than long-term memberships, and the difference between short- and long-term membership profits increases as competition becomes less intense.

What tariff do firms choose in equilibrium? To answer this question, we endogenize the pricing decision of firms between long- or short-term memberships in Section 5. That is, we consider a three-period game: in period 0, each firm chooses between long- or short-term memberships; and in period 1 and 2, the firms offer long- or short-term memberships as described above. We first study the asymmetric game in which one firm offers long-term and the other offers short-term memberships. We show that when firms use long-term memberships, they extract a larger share of consumer surplus in period 1 with the membership fee than when they use short-term memberships. We show that firms have incentives to deviate from the symmetric long-term membership game and instead offer short-term memberships. The firm that deviates to short-term memberships has more instruments to extract consumer surplus in period 2 and can more efficiently distribute consumer surplus extraction across the two periods. As a result, the firm can capture a higher market share in period 1. Finally, we show that firms do not have incentives to deviate from the symmetric short-term membership game. Thus, short-term membership is a Nash equilibrium.

[12]Note that the way firms set up their tariffs in our two-period membership model is similar to the previous literature on behavior price discrimination (e.g., Fudenberg and Tirole [11]; Fudenberg and Villas-Boas [12]).
[13]In Subsection 6.3, we show that the unit price offered to the old customers is higher and the share of switchers (customers poached) is lower, compared with the case in which firms use LP in both periods but are allowed to discriminate based on purchase history.
[14]Note that these equilibrium tariffs are similar to the ones used by online grocery delivery services (like *Amazon Fresh* and *Instacart*), which usually set low unit prices at each period and discriminate only with their membership fee. See also Table 1.

We extend the analysis of our benchmark two-period membership model in different directions in Section 6. First, we allow firms to offer long-term contracts that promise to supply products at a constant unit price in both periods. In equilibrium, firms set the long-term marginal price equal to the marginal cost and extract surplus through long-term contract membership fees, which are higher than the standard membership fees. When firms offer long-term memberships with long-term contracts with the unit price, captive demand of old customers in period 2 decreases (by those who accept the long-term contract in period 1), decreasing the overall profits. Second, we consider a model in which firms are not allowed to price-discriminate based on purchase history (i.e., between old and new customers) with their unit price. However, they are allowed to charge (subsidize) membership fees. We show that marginal cost pricing is not an equilibrium in period 2; instead, firms offer a subsidy proportional to the monopoly profit function with the fixed fee (i.e., negative fixed fees).[15] Third, we compare our model benchmark membership model, with a standard linear pricing model with no membership fees.

Contribution to the Literature. This article contributes to the literature on several inter related areas. Our primary contribution applies to the *competitive price discrimination* literature (e.g., Armstrong and Vickers [2], Rochet and Stole [18], Hoernig and Valletti [14], Yang and Ye [26], Armstrong and Vickers [3], and Tamayo and Tan [21]). First, from the literature on "static" models, it is well known that the profit-maximizing method of extracting surplus from consumers when firms charge membership fees and unit prices is to set the unit price equal to the marginal cost and to extract surplus through the fixed fee.[16] However, firms in membership business models deliver a flow of goods and services to their customers for different periods. Thus, previous predictions for the static models may not hold, given that firms extract surplus not only through membership fees but also through the unit price in both periods. Second, when firms use long-term membership fees, there is an important asymmetry in the competition for new and old customers: firms offer their old customers a unit price (LP) and a membership fee and a unit price to the new customers (2PT).

Moreover, Armstrong and Vickers [2], study competitive nonlinear pricing when consumers are differentiated *á la* Hotelling, have private information about their tastes for quality, and single home. They show that when firms are symmetric, in equilibrium, each firm offers a 2PT contract with a marginal price equal to the marginal cost. Rochet and Stole [18] show a similar result; firms offer a cost-plus-fee pricing schedule. In other words, there is an efficient quantity (or quality) provision supported by the marginal-cost-based 2PTs.[17] The analysis

[15]Most of the wireless carriers companies, like *Verizon*, *T-Mobile*, and *At&T*, offer the same monthly plan to their new and current customers, but discriminate with their membership fees.
[16]See, for example, Armstrong and Vickers [2], Rochet and Stole [18], and Tamayo and Tan [21].
[17] Armstrong and Vickers [3] generalize the model in Armstrong and Vickers [2] by assuming that consumers are allowed to buy from both firms or from just one (i.e., multi-home) and find that in equilibrium, firms offer

of general nonlinear pricing for firms competing in a two-period game is complicated. Our paper studies whether the equilibrium outcome is still efficient when firms use long- or short-term memberships—a smaller strategy space. Although we consider a smaller strategy space (dynamic memberships instead of dynamic nonlinear pricing), it is a first step to infer about the general case with dynamic nonlinear pricing schedules.

Another related literature pertains to behavioral price discrimination and customer recognition. In terms of the model setup, our work comes close to the seminal article of Fudenberg and Tirole [11], who study a two-period duopoly model with forward-looking firms and customers.[18] Other related literature on payoff-relevant history pertains to switching costs, pioneered by Chen [5], who analyzed a two-period duopoly model in which switching costs are heterogeneous, generating ex-post differentiation.[19] In general, both branches of the literature conclude that, in equilibrium, firms offer preferential pricing to new customers and charge higher prices to old customers.[20] Our membership model shares some of these results,

marginal-cost pricing (see also Armstrong [1]). Yang and Ye [26] consider a model similar to Armstrong and Vickers [2] and Rochet and Stole [18] but assume that the lowest-type consumer covered (in the market) is endogenously determined (i.e., consumers are not fully covered on the vertical dimension). Yin [27] considers a shipping cost model of 2PT competition in which the transportation cost interacts with the quantity, and consumers have homogeneous tastes for quality. He shows that marginal prices are equal to marginal cost if and only if the demand of the consumer who is indifferent between buying the i-good and the j-good for $i \neq j$ (i.e., the marginal consumer) is equal to the average demand. Hoernig and Valletti [14] consider a model where consumers are horizontally differentiated, *á la* Hotelling and mix goods offered by two firms. The authors show that when both firms use 2PTs, marginal-cost pricing is an equilibrium if and only if both firms are located at the same spot.

[18]The literature on behavior-based price discrimination extends in different directions. Esteves and Reggiani [10] extend the Fudenberg and Tirole [11] model and assume that consumers have constant elasticity demand, but their model considers only linear pricing. Villas-Boas [25] extends Fudenberg and Tirole [11] model to an infinite model with overlapping generations. Chen [6] assumes that firms are asymmetric where one firm has a stronger market position than its competitors. Esteves [9] extends Fudenberg and Tirole [11] and assumes that firms invest in advertising to generate awareness. Pazgal and Soberman [17] show that profits increase if only one firm uses past purchase information. Chen [7] studies whether firms have incentives to share information on customers' past purchase behavior and shows that the effect on consumers of doing so depends on the relative magnitude of the prices in the substitute goods market and the complementary goods market.

[19]Taylor [22] extends Chen's model to allow competition between more than two firms, an arbitrary number of periods, and persistent consumer heterogeneity. For a comprehensive review, see Fudenberg and Villas-Boas [13] and, more recently, Villas-Boas [24].

[20]Shin and Sudhir [20] and Chen and Pearcy [8] study a model in which brand preferences change over the two periods. The first paper shows that sufficient heterogeneity in the taste for quality of the consumer, and stochastic preferences are a key ingredient in observing firms that reward old, best-value customers. The latter shows that firms still offer lower prices to their rivals' customers when commitment to future prices is infeasible; however, consumer loyalty is rewarded when firms are allowed to commit to future prices. Caillaud and Nijs [4] provide a different theoretical explanation for why a firm may reward current customers; it comes from the reciprocity of the firms' incentives to acquire information from new customers and extract surplus from their loyal customers, and to reward their new customers. As Taylor [22] pointed out, there are differences between both types of models (i.e., behavioral price discrimination and switching cost models): if consumers in period 2 experience a large taste shock in regard to the firm, it may be efficient for them to switch. However, in switching cost models with homogeneous goods, changing suppliers is never efficient.

but there are also notable differences. We assume that consumers have elastic demands, and that firms charge a membership fee that allows consumers to buy multiple products at a given unit price in both periods. Equilibrium prices and consumer welfare depend on the structure of tariffs used by the firms; namely, long-term versus short-term memberships. We extend this model setup and endogenize the tariff decision of firms between long- or short-term memberships in a stage 0 of the game. In the extension of the benchmark model, we study how commitment to setting the unit price may affect the equilibrium profits of the firms. Finally, we study how the equilibrium pricing strategy changes if firms are not allowed to price-discriminate based on purchased history with the unit price. These new features provide a better setting to study the recent increases of membership business models.

2 Model

Two firms, A and B, offer horizontally differentiated products to a population of consumers, competing over two periods, 1 and 2. Both firms can produce their products at a constant marginal cost, c. We adopt a Hotelling type horizontal differentiation model with consumers uniformly located on the unit interval $[0,1]$. That is, consumers' preferences for the products can be represented by $u(q_A) - tx$, if they buy from firm A and $u(q_B) - t(1-x)$ if they buy from B, where $x \in [0,1]$ is the distance to firm A, $1-x$ the distance to firm B, and t is the transportation cost per unit of distance at each period, measuring the degree of horizontal product differentiation. We assume that consumers' preferences remain constant for both periods. In each period, consumers either buy all products from one firm or the other, or they consume from an outside option, u_0.

The next assumption characterizes the set of utility functions considered here.

Assumption 1. *The utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, satisfies $u(0) = 0$, $u''(\cdot) < 0$, $u'(0) > c$, and there exists a unique $q^e > 0$ such that $u'(q^e) = c$.*

The focus of this paper is on a membership (subscription) model in which consumers pay a membership fee (lump-sum fee), F_i, that allows them to buy products/services at a marginal price, p_i, for $i = A, B$. We define the set of feasible unit prices for both periods and both firms as $\mathcal{P}$. Given (p_i, F_i) in a given period, a consumer decides to buy $q_i : \mathcal{P} \to \mathbb{R}_+$ units from firm $i \in \{A, B\}$, where

$$q_i(p_i) = \arg\max_{q_i \in \mathbb{R}_+} \{u(q_i) - p_i q_i\}.$$

The aggregate utility $U_i(p_i, F_i)$ is

$$U_i(p_i, F_i) \equiv v(p_i) - F_i,$$

where $v(p_i)$ is the indirect utility "offered" by firm i, defined by[21]

$$v(p_i) \equiv \max_{q_i \in \mathbb{R}_+} \{u(q_i) - p_i q_i\}.$$

To simplify the analysis, we focus on the case of full market coverage, in which all consumers buy from at least one firm $i \in \{A, B\}$ and both firms sell strictly positive quantities in both periods. Note that (A1) implies that the buyer's demand function, $q(p_i)$, and monopoly profit function, $\pi(p_i)$, are continuously differentiable and that $q(p_i)$ is strictly decreasing on p_i.

Let $\mu_i(p_i) \equiv -\frac{q(p_i)}{q'(p_i)}$ to be the inverse of the quasi-elasticity of demand, where $q'(p_i) \equiv \frac{\partial q(p_i)}{\partial p_i}$.

Assumption 2. *$\mu'(p_i) < 1$.*[22]

The monopoly profit function

$$\pi(p_i) = q(p_i)(p_i - c)$$

is single-peaked in p_i under (A2), and hence there is a unique optimal monopoly price $p_i^m \in \mathcal{P}$ defined by $\frac{\partial \pi^i(p_i)}{\partial p_i} = 0$. We suppose that (A1) and (A2) are satisfied for the rest of the paper.

We assume that firms observe the first-period actions of their own consumers. Thus, the price offered by firm $i \in \{A, B\}$ in period 2 may depend on the consumer's purchase history. That is, firms may discriminate between "old" customers (who bought from firm i in period 1) and "new" customers (who bought from its rival in period 1) in period 2.[23]

We assume that consumers are forward-looking while firms cannot commit to a particular price level in period 2. Note that if consumers are naive and firms do not discriminate between new and old customers in period 2, in equilibrium, both firms charge marginal-cost-based 2PT in both periods (see, for example, Tamayo and Tan [21]). Finally, we assume that firms and consumers have a common discount factor $\delta \in [0, 1]$.

2.1 Long-Term Membership. In the benchmark model, we suppose that both firms charge a membership fee and a unit price (p_i, F_i) in the first period. The membership fee allows consumers to purchase from the same firm for the two periods; that is, consumers do not need to pay the fee again in the second period if they decide to buy from the same firm they purchased from in period 1. In the second period, each firm charges a single marginal

[21] Note that by Roy's identity, the indirect utility function, $v(p_i)$, satisfies $q(p_i) = -\partial v(p_i)/\partial p_i$.

[22] Tamayo and Tan [21] use a similar assumption in a 2PTs static model. Armstrong and Vickers [2] also have a similar assumption: They assume $\sigma'(u) \le 0$ where $\sigma(p) = -\frac{q'(p)}{q(p)}(p - c)$ for $u = v(p)$. The function $\sigma(p)$ represents the elasticity of demand expressed in terms of the mark up $(p - c)$ instead of the price p. Furthermore, as $p \to c$, $\sigma(p) \to 0$ and as $p \to p^m$, $\sigma(p) \to 1$. It follows that $\mu'(p) < 1$ implies that $\sigma'(u) \le 0$.

[23] Hereafter, we use subscript "n" ("o") to denote the price offered to the new (old) customers in period 2. Similarly, the superscript "2" denotes the price offered in period 2.

price to its old customers, $p^2_{i,o}$, and a membership fee and a marginal price to those who purchased from its rival in period 1 (new customers), $\left(p^2_{i,n}, F^2_{i,n}\right)$ for $i \in \{A, B\}$. Thus, in the second period, firms discriminate between new and old customers with both the marginal price and the membership fee.

Consumers in period 2 decide whether to buy from the same firm they bought from in period 1, or switch to the competitor. Since consumers are forward-looking (i.e., are not myopic), in period 1, they decide to buy from the firm that provides them the highest expected returns. Thus, the solution concept used is Perfect Bayesian Equilibrium.

Revealed-preference argument implies that for any pair of first-period prices, there is a cut-off, x^*, such that all consumers with $x < x^*$ buy from firm A in the first period, and all consumers with $x > x^*$ buy from firm B. Call the space between 0 and x^* as the *turf* of firm A and everything to the right of x^* as the *turf* of firm B.

Period 2. Let us start by describing the problem on A's turf in period 2 (i.e., the set of $x \in [0, 1]$ such that $x \leq x^*$). The problem of both firms on B's turf is equivalent, so we omit it. The problem of firm A on its own turf is

$$\max_{p^2_{A,o}} \quad s_A\left(p^2_{A,o}, p^2_{B,n}, F^2_{B,n}\right)\pi\left(p^2_{A,o}\right), \tag{2.1}$$

where $s_A\left(p^2_{A,o}, p^2_{B,n}, F^2_{B,n}\right) = \min\left\{x^*, \frac{1}{2} + \frac{v\left(p^2_{A,o}\right) - v\left(p^2_{B,n}\right) + F^2_{B,n}}{2t}\right\}$. Similarly, firm B charges a membership fee and a marginal price to consumers on A's turf (new customers), so the problem of firm B is

$$\max_{p^2_{B,n}, F^2_{B,n}} \quad \left(x^* - s_A\left(p^2_{A,o}, p^2_{B,n}, F^2_{B,n}\right)\right)\left(\pi\left(p^2_{B,n}\right) + F^2_{B,n}\right). \tag{2.2}$$

Period 1. Consumers located at x^* must be indifferent between buying from firm A in period 1 (pay the membership fee F_A and the marginal price p_A) and then switching and paying the fee, $F^2_{B,n}$, and the marginal price, $p^2_{B,n}$, to firm B in period 2, or buying from firm B in period 1 (pay the membership fee F_B and the marginal price p_B) and then buying from firm A at a marginal price, $p^2_{A,n}$, and a membership fee, $F^2_{A,n}$, in period 2.[24] The tariffs in the second period depend on x^*, and the market share of firm A in the first period depends on the first-period tariffs $\alpha \equiv (p_A, F_A, p_B, F_B)$. Thus, x^* is implicitly defined by

[24]Note that if the share of switchers is positive, it is not possible that a consumer located at x^* buys from the same firm in both periods, which implies that the rival firm provides a higher utility to new customers than the current firm.

$$x^{*}=\frac{1}{2}+\frac{v\left(p_{A}\right)-v(p_{B})+F_{B}-F_{A}+\delta\left(v\left(p_{B,n}^{2}\right)-v\left(p_{A,n}^{2}\right)\right)+\delta\left(F_{A,n}^{2}-F_{B,n}^{2}\right)}{2t\left(1-\delta\right)}. \tag{2.3}$$

Note that if firms are symmetric, in equilibrium, it follows that $x^{*}=\frac{1}{2}$. But if firms are not symmetric, we would have to solve a system of equations and verify the existence and uniqueness of x^{*}.[25]

Now, the overall problem of firm A in the first period is

$$\max_{p_{A},F_{A}} \underbrace{x^{*}\left(\alpha\right)\left(\pi\left(p_{A}\right)+F_{A}\right)}_{(1)}+\underbrace{\delta s_{A}\left(p_{A,o}^{2},p_{B,n}^{2},F_{B,n}^{2}\right)\pi\left(p_{A,o}^{2}\right)}_{(2)}$$
$$+\delta\underbrace{\left[s_{B}\left(p_{B,o}^{2},p_{A,n}^{2},F_{A,n}^{2}\right)-x^{*}\left(\alpha\right)\right]\left(F_{A,n}^{2}+\pi\left(p_{A,n}^{2}\right)\right)}_{(3)}.$$

Note that firm A's overall objective function depends on three terms: (1) is equal to the share of consumers who buy from firm A in period 1, x^{*}, multiplied by the membership fee and the monopoly profit function; (2) is equal to the market share of customers who buy from firm A in period 1 and buy again from firm A in period 2, multiplied by the monopoly profit function; and (3) the share of switchers, meaning those who buy from firm B in period 1 and then buy from firm A in period 2, multiplied by the membership fee and the monopoly profit function charged to new customers.

We can interpret the access to firm i as product 1 with a price given by the fixed fee, F_{i}, and the real product offered by firm i as product 2, with a price equal to p_{i}. Then, x^{*} is the demand for product 1 of firm A in period 1. Consumer's participation incentives can be described by the marginal rate of substitution of the demand for access (MRSA) between p_{A} and F_{A} introduced by Tamayo and Tan [21].[26] The MRSA between p_{A} and F_{A} in period 1 is

$$\mathrm{MRSA}_{A}^{1}\equiv\tfrac{\partial x^{*}}{\partial p_{A}}/\tfrac{\partial x^{*}}{\partial F_{A}}. \tag{2.4}$$

Similarly, the second-period demand for product 1 from new customers of firm A is $s_{B}\left(p_{B,o}^{2},p_{A,n}^{2},F_{A,n}^{2}\right)-x^{*}$ with price equal to $p_{A,n}^{2}$, so that the MRSA between $p_{A,n}^{2}$ and $F_{A,n}^{2}$ is

[25] Note that in single unit demand models, it is feasible to get an explicit solution for x^{*}. In our model, consumers have general elastic demands, then, x^{*} is implicit defined by (2.3) and depends of first- and second-period tariffs.

[26] As pointed out by Tamayo and Tan [21], the MRSA is a more general description than the demand of the marginal consumer that has been identified in the literature regarding monopoly two-part tariffs (e.g., Schmalensee [19] and Varian [23]).

$$\text{MRSA}^2_{A,n} \equiv \frac{\partial\left(s_B\left(p^2_{B,o}, p^2_{A,n}, F^2_{A,n}\right) - x^*\right)}{\partial p^2_{A,n}} \Big/ \frac{\partial\left(s_B\left(p^2_{B,o}, p^2_{A,n}, F^2_{A,n}\right) - x^*\right)}{\partial F^2_{A,n}}. \tag{2.5}$$

The MRSA for firm B can be defined analogously.

2.2 Short-Term Membership. If memberships are valid for one period (short-term memberships), consumers need to *renew* their memberships in period 2. Under this arrangement, customers need to pay the membership fee in both the first and second periods. We assume that firms are allowed to discriminate with their marginal price and membership fee based on purchase history. Thus, they offer a membership fee and a marginal price to both old and new customers. In the second period, the problem of firm A on its own turf is now

$$\max_{p^2_{A,o}, F^2_{A,o}} \quad s_A\left(p^2_{A,o}, F^2_{A,o}, p^2_{B,n}, F^2_{B,n}\right)\left(\pi\left(p^2_{A,o}\right) + F^2_{A,o}\right), \tag{2.6}$$

where $s_A\left(p^2_{A,n}, F^2_{A,n}, p^2_{B,o}, F^2_{B,o}\right) \equiv \frac{1}{2} + \frac{v\left(p^2_{A,n}\right) - v\left(p^2_{B,n}\right) - F^2_{A,o} + F^2_{B,n}}{2t}$. Similarly, firm B charges a membership fee and a marginal price to consumers on A's turf, so the problem of firm B is

$$\max_{p^2_{B,n}, F^2_{B,n}} \left(x^* - s_A\left(p^2_{A,o}, F^2_{A,o}, p^2_{B,n}, F^2_{B,n}\right)\right)\left(\pi\left(p^2_{B,n}\right) + F^2_{B,n}\right). \tag{2.7}$$

The problem of firm A in the first period is

$$\max_{p_A, F_A} \quad x^*(\alpha)\left(\pi\left(p_A\right) + F_A\right) + \delta s_A\left(p^2_{A,o}, F^2_{A,o}, p^2_{B,n}, F^2_{B,n}\right)\left(\pi\left(p^2_{A,o}\right) + F^2_{A,o}\right) \tag{2.8}$$

$$+\delta\left[s_B\left(p^2_{B,o}, F^2_{B,o}, p^2_{A,n}, F^2_{A,n}\right) - x^*(\alpha)\right]\left(F^2_{A,n} + \pi\left(p^2_{A,n}\right)\right).$$

The interpretation of firms A's overall objective function is similar to the interpretation provided above for the long-term membership model. The MRSA of the demand for access between the price and fixed fee can be analogously defined for period 1 and period 2. Notice that in this game, there is also a MRSA for old customers in the second period, since they are being offered a 2PT.

3 Long-Term Membership

In this section, we study our benchmark model and show that the membership fee offered to new customers in the second period is lower than the fee offered in the first period. Similarly, the marginal price offered to old customers is higher than the price offered to new customers in both periods.

3.1 Membership Model. In period 2, firm B will not capture the entire market on A's turf; then, from the first-order conditions of (2.1) and (2.2), the fixed fee charged by firm B in period 2 to new customers is

$$F_{B,n}^2 = \frac{t\left(2x^* - 1\right) - v\left(p_{A,o}^2\right) + v\left(p_{B,n}^2\right) - \pi\left(p_{B,n}^2\right)}{2}, \tag{3.1}$$

$p_{B,n}^2 = c$, and $p_{A,o}^2$ is implicitly defined by

$$t\left(1 + 2x^*\right) - v\left(c\right) = \psi\left(p_{A,o}^2\right), \tag{3.2}$$

where $\psi\left(p\right) \equiv 2\phi\left(p\right) - v\left(p\right)$ and $\phi\left(p\right) \equiv \frac{q(p)\pi(p)}{\pi'(p)}$. The next proposition summarizes the equilibrium in the second period.

Proposition 1. *Given the existence of a first-period cutoff, there exists a unique interior equilibrium in period 2 in which:*

(i) *for $t > 0$, small, there exists $\underline{x} < \frac{1}{2}$ and $\bar{x} > \frac{1}{2}$ such that for $x^* \in [\underline{x}, \bar{x}]$, each firm charge a marginal price to the old customers on its own turf $p_{i,o}^{2*} > c$ defined by (3.2), and a membership fee defined by (3.1) and a marginal price equal to the marginal cost to new customer for $i \in \{A, B\}$;*[27]

(ii) $s_A\left(p_{A,o}^{2*}, p_{B,n}^{2*}, F_{B,n}^{2*}\right) < x^*$.

When firms use membership fees, the optimal strategy in period 2 is to charge a positive membership fee (defined by (3.1)) and a marginal price equal to the marginal cost to new customers. Old customers do not need to pay the subscription fee again; thus, when price discrimination is allowed, firms set a higher marginal price than the one offered to new customers. Note that turfs are "independent" markets. The game on each turf is closely related to a static asymmetric two-part tariffs game (2PTs); one firm uses a marginal price (charged to its old customers) and the other firm uses 2PTs (to its new customers). Given that consumers have homogeneous taste preferences, the firm that uses 2PTs will charge marginal-cost-based 2PTs independently of its rival's marginal price. This result is consistent with the literature on price discrimination on a static framework.[28]

From (3.1) and the proof of Proposition 1, as $x^* \to \underline{x}$, $F_{B,n}^2 \to 0$ and the market share for firm B decreases up to a point in which the indirect utility provided by firm B would not be enough to compensate the transportation cost. The following corollary provides some comparative statics with respect to the cutoff value x^*:

[27] By t small, we mean $t < v(c)$.

[28] See, for example, Tamayo and Tan [21] for a model of competition with 2PTs.

Corollary 1. *In any interior equilibrium,*

(i) $\frac{\partial p_{A,o}^{2}}{\partial x^{*}} > 0$ *and* $\frac{\partial F_{B,n}^{2}}{\partial x^{*}} > 0$;

(ii) $\frac{\partial p_{B,o}^{2}}{\partial x^{*}} < 0$ *and* $\frac{\partial F_{A,n}^{2}}{\partial x^{*}} < 0$.

Corollary 1 shows that as the market for new consumers for firm B increases (turf A increases; i.e., x^{*} increases), the membership fee for the new customers increases. The analysis is similar for $p_{A,o}^{2}$ and equivalent on B's turf. That is, as the market for new consumers for firm B increases (x^{*} increases), firms compete less aggressively on A's turf and more aggressively on B's turf. Intuitively, if firm A is larger (has more consumers in the first period), then in the second period, firms compete more aggressively for current firm B customers and less aggressively for current firm A consumers.

The following proposition summarizes the overall equilibrium of the game.

Proposition 2. *There is a unique symmetric equilibrium in which firms*

(i) *in the second period, charge a marginal price* $p_{o}^{2*} = \psi^{-1}\left(2t - v(c)\right) > c$ *to the old customers and marginal cost-based membership fee to the new customers (i.e.,* $p_{n}^{2*} = c$*) with a membership fee equal to* $F_{n}^{2*} = \frac{v(c) - v\left(p_{o}^{2*}\right)}{2}$*, where* $\psi\left(p\right) \equiv 2\phi\left(p\right) - v\left(p\right)$;

(ii) *in the first period, charge marginal-cost-based membership fee (i.e.,* $p = c$*) and*

$$F^{*} = t + \delta\frac{t \cdot q\left(p_{o}^{2*}\right)}{2\phi'\left(p_{o}^{2*}\right) + q\left(p_{o}^{2*}\right)} + \delta\frac{\phi'\left(p_{o}^{2*}\right) + q\left(p_{o}^{2*}\right)}{2\phi'\left(p_{o}^{2*}\right) + q\left(p_{o}^{2*}\right)}\left(v\left(c\right) - v\left(p_{o}^{2*}\right) - \pi\left(p_{o}^{2*}\right)\right) > 0;$$

(iii) $F^{*} > F_{n}^{2*}$;

(iv) $s_{A} = \frac{1}{2} + \frac{v\left(p_{o}^{2*}\right) - v(c)}{4t} < \frac{1}{2}$ *and* $s_{B} = \frac{1}{2} + \frac{v(c) - v\left(p_{o}^{2*}\right)}{4t} > \frac{1}{2}$.

From Proposition 1 and 2, we know that in the second period, firms offer a unit price to old customers above the marginal cost, and a marginal-cost-based membership fee to new customers. Thus, the equilibrium pricing strategy for them to attract and extract surplus from new customers is to set the marginal price equal to the marginal cost and extract surplus through the membership fee. This extra pricing instrument allows firms to extract surplus more efficiently than the LP game, as we show in Section 6; they charge a membership fee to new customers that is proportional to the difference of the efficient surplus and the surplus offered by the rival firm, $v\left(c\right) - v\left(p_{o}^{2}\right)$. Note that the membership fee is linear in the profit function of firm i, so it does not depend directly on the curvature of the demand. Similarly, firms offer cost-based membership fees to their new customers in period 1, but the membership fee to new customers in period 2 is lower than the fee offered in period 1.[29]

[29]This result is similar to that in other literature of behavior-based priced discrimination (e.g., Fudenberg and Tirole [11]).

Finally, in equilibrium, the share of switchers is proportional to the difference of the efficient surplus and the surplus offered to the old customers.[30]

Why do firms use cost-based membership fees in period 2? As we mentioned before, in period 2, each firm's turf can be considered as a different market in which one firm offers LP and the other firm 2PTs. Note that from (2.5) it follows that the MRSA between the instruments $p^2_{A,n}$ and $F^2_{A,n}$ is

$$\mathrm{MRSA}^2_{A,n} = q(p^2_{A,n}). \tag{3.3}$$

Thus, the MRSA between $p^2_{A,n}$ and $F^2_{A,n}$ is equal to the demand for product 2 by the new customers. Moreover, note that the MRSA between $p^2_{A,n}$ and $F^2_{A,n}$ is equal to the *average demand*.[31] The average demand is equal to the unconditional demand divided by the market share. It follows that there are no gains from increasing (or decreasing) the marginal price, $p^2_{A,n}$, above (or below) the marginal cost to increase the number of participating consumers. Instead, the firm is better off extracting consumer surplus just with its fixed fee. From Corollary 1 in Tamayo and Tan [21], it follows that any pure strategy Nash equilibrium in 2PTs in the second period involves marginal-cost-pricing; that is, $p^{2*}_{A,n} = c$.

What about period 1? We need to modify our analysis to take into account that first-period tariffs affect second-period prices and market shares. Using the Implicit Function Theorem, the first-order conditions of each firm, and equation (2.3), we show that changes in the market share in period 1, x^*, due to changes in p_A are proportional to changes in x^* due to changes in the fixed fee, F_A, and the ratio of these changes is equal to $q(p_A)$. In other words, from (2.4) it follows that the MRSA between the two instruments p_A and F_A is

$$\mathrm{MRSA}^1_A = q(p_A), \tag{3.4}$$

which is equal to the average demand in period 1.[32] Note that again firms do not have incentives to increase (or decrease) the marginal price above (or below) the marginal cost. First-period tariffs (membership fee and marginal price) affect new and old customers proportionally and equal to $q(p_A)$. Thus, any pure strategy Nash equilibrium in 2PTs in the first period involves marginal-cost-pricing (i.e., $p^*_A = c$).

[30]Customers are forward-looking and correctly anticipate that firms will compete for them in the second period, offering them smaller prices. The latter, combined with the fact that firms know which users are new (or old) in the second period, explains why there is a measure of positive switchers in the second period, even though firms are symmetric and play symmetric strategies in equilibrium.

[31]The average demand for new customers in the second period is equal to the unconditional demand, $\left(s_B\left(p^2_{B,o}, p^2_{A,n}, F^2_{A,n}\right) - x^*(\alpha)\right) q(p^2_{A,n})$, divided by the market share, $s_B\left(p^2_{B,o}, p^2_{A,n}, F^2_{A,n}\right) - x^*(\alpha)$, which is equal to $q(p^2_{A,n})$.

[32]Similarly, the average demand of firm A in the first period is equal to the unconditional demand, $x^* q(p_A)$, divided by the market share, x^*.

Proposition 2 is also related to Mathewson and Winter [15] for goods that are *strongly complementary* in demand. The demand for product 1 is the market share of firm B's product, $x^* - s_A\left(p^2_{A,o}, c, F^2_{B,n}\right)$, and the demand for product 2 is the market share multiplied by the individual demand for the product, $[x^* - s_A\left(p^2_{A,o}, c, F^2_{B,n}\right)]q\left(p^2_{B,n}\right)$. Note that the ratio is independent of the subscription fee, $F^2_{B,n}$; hence the two products are strong complements. Using Proposition 2 in Mathewson and Winter, we can conclude that firm B's profits are maximized by setting $p^2_{B,n} = c$, *independently* of $p^2_{A,o}$. A similar logic allows us to conclude why the marginal price is equal to the marginal cost to the new customers for firm A.

From the literature on competitive price discrimination (e.g., Armstrong and Vickers [2], Rochet and Stole [18] and Tamayo and Tan [21]), we may expect that when consumers have homogeneous taste preferences, firms charge cost-based membership fees. Note, however, that the previous literature on 2PTs considers only static games; thus this result may not be obvious in a dynamic setting.

These results may explain the empirical regularities observed in the wire and cable market. Typically, customers need to pay a membership fee and a marginal price at the beginning of the contract, and in the next period (usually a year), firms offer old customers a higher marginal price than the marginal price and fee offered to new clients who switch from a different company. For example, *DirecTV* and *AT&T* charge an initial membership fee for a two-year contract and offer a very low unit price (monthly rate) for the first year, but do not commit to a specific price in period 2 (See figure 7.1). Moreover, the following condition was specified in the *DirecTV* offer: "New approved residential customers only."[33] Cell phone company *Sprint* also charges a membership fee and offers a cheaper monthly plan rate specific to new customers who were previously enrolled with other carriers.[34] There are only two periods in our model, so we need to assume that the new customers who switch were previously enrolled in a different company.

4 Short-Term Membership

After analyzing the effects of long-term memberships, we now study short-term ones. Compared with the previous model, here consumers need to "renew" their subscription in the second period, even if they purchase from the same firm they subscribed to in the first period. The assumptions of the previous section remain valid in this section.

From the first-order conditions, in period 2, it follows that marginal-cost-based 2PT is a unique equilibrium; that is, for any marginal price and membership fee of firm B, firm A sets

[33] Similarly, in the recent merger of *Time Warner Cable* and *Spectrum*, new offers were made exclusively to new customers. The offers available online had the following information (or caveat) for old customers: "Offers are valid for a limited time only, to qualifying residential customers who have not subscribed to applicable services within the previous 30 days and have no outstanding obligation to TWC."

[34] In March 2017, *Sprint* offered deeply discounted prices on its unlimited plan only to new customers who were previously enrolled with carriers such as *Verizon* or *T-Mobile*.

its marginal price equal to the marginal cost and extracts surplus through the membership fee. Following a similar argument, firm B uses marginal-cost-based 2PT on A's turf. Note that the problem is similar to a standard competitive 2PT model (see Tamayo and Tan [21]). In equilibrium,

$$F_{A,o}^2 = \frac{t\left(2x^* + 1\right)}{3}, \quad F_{B,n}^2 = \frac{t\left(4x^* - 1\right)}{3},$$

and $p_{i,o}^2 = p_{i,n}^2 = c$ for $i \in \{A, B\}$. Note that there exists an interior equilibrium with positive profits on each turf for both firms for $x^* \in \left[\frac{1}{4}, \frac{3}{4}\right]$. Similarly, note that $F_{i,o}^2 > F_{i,n}^2$ for $i = A, B$, for $x^* < 1$.

Let us now consider the first-period pricing and consumers' decisions. Given that consumers are not myopic and anticipate the firms' second-period pricing strategy, at an interior equilibrium, the type-x^* consumer is such that

$$x^* = \frac{1}{2} + 3 \cdot \frac{v\left(p_A\right) - v\left(p_B\right) + F_B - F_A}{2\left(3 + \delta\right) t},$$

where we use the fact that $F_{A,n}^2 - F_{B,n}^2 = \frac{4t(1-2x^*)}{3}$ and $p_{i,o}^2 = p_{i,n}^2 = c$ for $i \in \{A, B\}$. The following proposition characterizes the equilibrium.

Proposition 3. *There is a unique symmetric equilibrium in which* $x^* = \frac{1}{2}$, $s_A = \frac{1}{3}$, $s_B = \frac{2}{3}$, $F_o^2 = \frac{2t}{3}$, $F_n^2 = \frac{t}{3}$ *and* $F = \frac{(3+\delta)}{3} t$.

When consumers have to pay membership fees in both the first and second period, we get a model that is close to the *poaching* model proposed by Fudenberg and Tirole [11]. In the second period, firms offer efficient surplus to both types of customers, but they offer a lower membership fee to their new customers. Also, the membership fee in the first period is higher than the membership fees offered to both old and new customers in the second period (i.e., $F > F_o^2 > F_n^2$).

Note that the MRSA between the two instruments, the price and the fixed fee, is equal to the demand of firm's i product, $q\left(p_i\right)$. Thus, any pure strategy Nash equilibrium involves setting marginal prices equal to the marginal cost and to extracting surplus with the fixed fee.

The results of this section on short-term memberships may explain the empirical regularities observed in the booming online food ordering and delivery services (e.g., *Uber Eats*, *Doordash*, and *GrubHub*) and online grocery delivery services market (e.g., *Amazon Fresh* and *Instacart*).[35] Firms do not generally discriminate between old and new customers with their unit prices, although they usually provide discounts on their membership fees to new clients (see Figure 7.2). In the model presented here, in equilibrium, firms offer marginal-cost

[35] Other traditional examples are warehouse clubs like *Costco* and *Sam's Club*.

pricing in both periods and extract surplus through the fixed fees from both new and old customers. In equilibrium, firms do not discriminate between current and new clients with their marginal price, except by providing discounted membership fees to the new customers.

4.1 Long- versus Short-Term Memberships. When do firms can keep a larger share of their old customers? When do firms obtain higher equilibrium profits? We now compare different equilibrium outcomes for both models (long- versus short-term memberships). Let the subscripts l and s denote equilibrium outcomes for the long-term and short-term membership games, respectively. That is, let $s_{o,l}$ and $s_{o,s}$ denote the market share of old customers in period 2 for the long- and short-term game, respectively. The following corollary compares the market shares of both games.

Corollary 2. *For t small, the following three conditions are satisfied and equivalent:*

(i) $s_{o,s} \leq s_{o,l}\left(p_{o,l}^2, c, F_{n,l}^{2*}\right) < \frac{1}{2}$;
(ii) $s_{n,l}\left(p_{o,l}^2, c, F_{n,l}^{2*}\right) \leq s_{n,s} < \frac{1}{2}$;
(iii) $F_{n,l}^{2*} \leq F_{n,s}^{2*}$.

When firms offer long-term memberships, the share of switchers in period 2 is lower than when firms use short-term memberships. When consumers do not have to renew their subscription and buy from the same firm they bought from in period 1, they only have to pay a marginal price for the products, while the rival firm charges them a marginal price and a membership fee. That is, the rival firm has more tools to extract consumer surplus (price and membership fee), which allow it to compete less aggressively for a higher market share of new customers, whereas firms need to compete more aggressively for their old customers (i.e., protect their old customers) given that they only have one tool—the marginal price.

When firms offer short-term memberships, both firms offer 2PT to their old and new customers; thus, firms compete less aggressively for their old customers, and the share of switchers is higher. That is, both firms can extract more surplus from their old and new customers than in the case in which firms offer long-term memberships (i.e., both have more tools). Finally, note that there is a higher inefficiency due to the higher share of switchers when firms offer short-term memberships.

Let us denote by $\pi_l(t)$ and $\pi_s(t)$ the equilibrium profits of the long- and short-term membership games, respectively, which are defined as

$$\pi_l(t) \equiv \frac{1}{2}F_l^* + \delta s_{o,l}\left(p_{o,l}^{2*}, c, F_{n,l}^{2*}\right)\pi\left(p_{o,l}^{2*}\right) + \delta\left[\frac{1}{2} - s_{o,l}\left(p_{o,l}^{2*}, c, F_{n,l}^{2*}\right)\right]F_{n,l}^{2*} \tag{4.1}$$

and

$$\pi_s(t) \equiv \frac{1}{2}F_s^* + \delta s_{o,s}F_{o,s}^{2*} + \delta\left(\frac{1}{2} - s_{o,s}\right)F_{n,s}^{2*}. \tag{4.2}$$

The following proposition compares the equilibrium profits when firms offer long- versus short-term memberships.

Proposition 4. *For t small, $\pi_s(t) > \pi_l(t)$.*

From Proposition 4, it follows that firms obtain higher profits by offering short-term rather than long-term memberships. As competition becomes more intense (i.e., $t \to 0$), long-term and short-term prices and memberships tend to c and 0, respectively.[36] As the market becomes less competitive (i.e., t increases), firms offering short-term membership can extract consumer surplus from their old customers more efficiently than firms offering long-term memberships. That is, as t increases: (i) membership fees increase linearly, whereas the prices charged to old customers when firms offer long-term memberships depend on the curvature of the demand; (ii) it becomes more difficult for the rival firm to poach customers and charge them cost-based memberships. Thus, the ability to charge a membership fee to new and old customers becomes more important as the market becomes less competitive.

Finally, note that when firms use long-term memberships, they extract a larger portion of their profits in the first period than when firms use short-term memberships. Intuitively, firms have more instruments to extract consumer surplus in period 2 when they offer short-term memberships, which allow them to distribute consumer surplus extraction more efficiently across the two periods. In contrast, when firms use long-term memberships, firms are able to retain a larger share of its customers in period 2, but need to extract a larger portion of their profits in period 1.

5 Equilibrium

The previous section shows that both firms obtain higher profits offering short-term instead of long-term memberships. What tariff do firms choose in equilibrium? In this section, we answer this question by endogenizing the pricing decision of firms between long- or short-term memberships.

We consider the following three-period game. In period 0, each firm chooses between long- or short-term memberships. If firm i chooses long-term memberships, it charges a membership fee and a marginal price (p_i, F_i) in period 1; and in period 2, it charges a single marginal price to its old customers, $p^2_{i,o}$ and a membership fee and a marginal price to those who purchased from its rival in period 1 (new customers), $\left(p^2_{i,n}, F^2_{i,n}\right)$. If firm i chooses short-term memberships, it charges a membership fee and a marginal price (p_i, F_i) in period 1, and a marginal price and a membership fee to its new and old customers—$\left(p^2_{i,n}, F^2_{i,n}\right)$ and $\left(p^2_{i,o}, F^2_{i,o}\right)$, respectively, in period 2.

To understand what tariffs firms choose in equilibrium in period 0, we first study the asymmetric game in which one firm offers long-term and the other firm offers short-term

[36]That is, $p^{2*}_{o,l} \to c$ and $F^{2*}_{n,l}, F^*_l \to 0$ and $F^{2*}_{n,s}, F^*_s \to 0$, as $t \to 0$.

memberships. Next, we compare the profits that firms obtain in the asymmetric game with the profits in the two symmetric games (long- and short-term memberships) studied in Sections 3 and 4. We show that firms have incentives to deviate from the symmetric long-term membership game, and instead offer short-term memberships. Moreover, we show that firms do not have incentives to deviate from the symmetric short-term membership game. Thus, we conclude that a short-term membership is a Nash equilibrium.

5.1 Asymmetric Model. We assume that firm A offers short-term memberships while firm B offers long-term memberships. In the second period, on A's turf, firm A solves the problem (2.6) and firm B solves the problem (2.7). Similarly, on B's turf, firm B solves the analogue of problem (2.1) for firm B and firm A solves the analogue of problem (2.2) for firm A.[37] From Proposition 3, it follows that firm A sets its marginal price equal to the marginal cost to old and new customers and extract surplus with the fixed fee. Similarly, firm B offers marginal-cost-based memberships to its new customers, and offers a marginal price to its old customer defined as

$$p_{B,o}^{2*} = \psi^{-1}\left(t\left(3-2x^*\right)-v\left(c\right)\right), \tag{5.1}$$

where $\psi\left(p\right) \equiv 2\phi\left(p\right) - v\left(p\right)$.

In the first period, the type-x^* consumer is implicitly defined as

$$x^*\left(\alpha\right) = \frac{1}{2} + \frac{v\left(p_A\right) - v\left(p_B\right) + F_B - F_A + \delta\left(F_{A,n}^2 - F_{B,n}^2\right)}{2t\left(1-\delta\right)}, \tag{5.2}$$

where $\alpha = \left(p_A, F_A, p_B, F_B\right)$. The problem of Firm A in period 1 is

$$\begin{aligned}\max_{p_A,F_A} \quad & x^*\left(\alpha\right)\left(\pi\left(p_A\right)+F_A\right) + \delta s_A\left(p_{A,o}^2, F_{A,o}^2, p_{B,n}^2, F_{B,n}^2\right)\left(\pi(p_{A,o}^2)+F_{A,o}^2\right) \\ & + \delta\left[s_B\left(p_{B,o}^2, p_{A,n}^2, F_{A,n}^2\right) - x^*\left(\alpha\right)\right]\left(\pi(p_{A,n}^2)+F_{A,n}^2\right),\end{aligned} \tag{5.3}$$

and the problem of firm B is

[37] We mean that firm B solves the problem

$$\max_{p_{B,o}^2} \left(1 - s_B\left(p_{B,o}^2, p_{A,n}^2, F_{A,n}^2\right)\right)\pi\left(p_{B,o}^2\right),$$

where $s_B\left(p_{B,o}^2, p_{A,n}^2, F_{A,n}^2\right) = \frac{1}{2} + \frac{v\left(p_{A,n}^2\right) - v\left(p_{B,o}^2\right) - F_{A,n}^2}{2t}$, and firm A solves the problem

$$\max_{p_{A,n}^2, F_{A,n}^2} \left(s_B\left(p_{B,o}^2, p_{A,n}^2, F_{A,n}^2\right) - x^*\right)\left(\pi\left(p_{A,n}^2\right) + F_{A,n}^2\right).$$

$$\max_{p_B,F_B} \quad (1-x^*(\alpha))\left(\pi(p_B)+F_B\right)+\delta\left(1-s_B\left(p^2_{B,o},p^2_{A,n},F^2_{A,n}\right)\right)\pi\left(p^2_{B,o}\right) \tag{5.4}$$
$$+\,\delta\left[x^*(\alpha)-s_A\left(p^2_{A,o},F^2_{A,o},p^2_{B,n},F^2_{B,n}\right)\right]\left(\pi(p^2_{B,n})+F^2_{B,n}\right).$$

The following proposition characterizes the equilibrium of the asymmetric game.

Proposition 5. *For t small, there exists a unique equilibrium in which:*[38]

(i) *there exists a function $g:[0,1]\longrightarrow[0,1]$ and a unique cutoff $x^*>1/2$, such that $x^*=g(x^*)$;*

(ii-a) *in the second period, firm B charges a marginal price $p^{2*}_{B,o}>c$ determined by (5.1), and marginal cost-based membership fee to the new customers (i.e., $p^{2*}_{B,n}=c$) with a membership fee equal to $F^{2*}_{B,n}=\frac{1}{3}t\left(4x^*-1\right)$;*

(ii-b) *firm A charges a marginal cost-based membership fee to both types of consumers, i.e. ($p^{2*}_{A,o}=p^{2*}_{A,n}=c$) with a membership fees equal to $F^{2*}_{A,o}=\frac{1}{3}t\left(2x^*+1\right)$ and*

$$F^{2*}_{A,n}=\frac{1}{2}\left(t\left(1-2x^*\right)+v\left(c\right)-v\left(p^2_{B,o}\right)\right);$$

(iii) *in the first period, firms charge marginal cost-based membership fees i.e., $p^*_A=p^*_B=c$ and fees F^*_A and F^*_B determined by (A.44) and (A.45), respectively.*

First, note that because we cannot impose symmetry in this game, we need to show that x^*—implicitly defined by (5.2)—exists. In the proof of Proposition 5, we show that the existence of x^* follows from (5.2) and *Brouwers' fixed-point theorem*, and uniqueness follows from *Banach's fixed-point theorem*. Second, note that, as the reader may expect, x^* is diffeerent from $1/2$. In equilibrium, firm A has a larger market share in period 1 (i.e., $x^*>1/2$, where $[0,x^*]$ is the turf of firm A), as firm B charges a higher fixed fee and extracts a larger share of its profits in period 1 than firm A. Firm A has more tools to extract profits in period 2, thus, does not need to charge a high membership fee in period 1. Finally, Proposition 5 shows that both firms offer marginal-cost-based membership in period 1. In period 2, both firms charge their new customers a marginal price equal to the marginal cost and positive fixed fees, and firm A offers its old customers a cost-based membership whereas firm B offers a marginal price determined by (5.1).

The surprising result in the first period is again a consequence of the equality between the MRSA between the instruments p_i and F_i and the average demand $q(p_i)$ for $i\in\{A,B\}$. From the first-order conditions of (5.3) and (5.4) and from (5.2), we show in the proof of Proposition 5 that changes in the market share in period 1, x^*, due to changes in p_A, are proportional to changes in x^* due to changes in the fixed fee, F_A, and the ratio is equal

[38]In this proposition and in Corollary 3, by t small, we mean $t\in(0,t^*)$, where $t^*\in(0,v(c))$ and it is defined in the proofs of Proposition 5 and Corollary 3. To prove the uniqueness of the cutoff $x^*>1/2$, we need to assume that ${}^{-q'(c)}/_{q(c)}<6.4$.

to $q(p_A)$. Similarly, for firm B, the same results holds for the respective market share of the firm in period 1, $1 - x^*$. The following corollary characterizes firms' profits from both customers (new and old) and both periods.

Corollary 3. *For t small:*

(i) *firm A's profits are larger than firm B's profits, in period 1;*

(ii) *if $\delta > 27/41$, firm A' profits from old customers are larger than the respective firm B's profits;*

(iii) *firm B's profits from new customers are larger than the respective firm A's profits;*

(iv) *overall A's profits are larger than overall B's profits.*

Part (i) of Corollary 3 shows that although firm B charges a higher membership fee than firm A in period 1 (i.e., $F_B^* > F_A^*$), firm A's profits are larger than firm B's profits (i.e., $x^* F_A^* > (1 - x^*) F_B^*$), since firm A has a larger market share in period 1. Part (ii) follows from two observations: (a) firm A's market share of old customers who buy again from firm A is larger than firm B's market share of old customers who return to firm B (i.e., $s_A \left(c, F_{A,o}^{2*}, c, F_{B,n}^{2*}\right) > 1 - s_B \left(p_{B,o}^{2*}, c, F_{A,n}^{2*}\right)$ for δ large enough), and (b) the profits obtained through fixed fee $F_{A,o}^{2*}$ are larger than the ones that B obtains from $\pi(p_{B,n}^{2*})$ (i.e., $F_{A,o}^{2*} > \pi(p_{B,n}^{2*})$). Part (iii) shows that since the share of new customers is larger for firm B, it charges them a marginal price equal to the marginal cost and a higher fixed fee than firm A (i.e., $F_{B,n}^{2*} > F_{A,n}^{2*}$). Also, the share of new customers who switch from firm A to firm B is larger than the share of new customers for firm A (i.e., $x^* - s_A \left(c, F_{A,o}^{2*}, c, F_{B,n}^{2*}\right) > s_B \left(p_{B,o}^{2*}, c, F_{A,n}^{2*}\right) - x^*$). Finally, firm A's overall profits are larger than firm B's.

5.2 Equilibrium Tariff. Recall that $\pi_l(t)$ and $\pi_s(t)$, given by (4.1) and (4.2), respectively, are the profits that firms obtain in equilibrium when both firms choose long- or short-term memberships, respectively.

Proposition 6. *For t small, firms choose short-term memberships in equilibrium.*

Let $\pi_{A,s}(t)$ and $\pi_{B,l}(t)$ be the equilibrium profits if firm A offers short-term and firm B offers long-term memberships.[39] Remember that from Proposition 4 , we know that for t small, firms obtain higher profits if both firms offer short-term rather than long-term memberships (i.e., $\pi_s(t) > \pi_l(t)$). Proposition 6 shows that for t small, $\pi_{A,s}(t) > \pi_l(t)$ and $\pi_s(t) > \pi_{B,l}(t)$. In other words, Proposition 6 shows that when both firms offer long-term memberships, each firm has incentives to deviate in period 0 to short-term memberships. Similarly, Proposition 6 shows that firms do not have incentives to deviate from short-term memberships in period 0. It follows that the three-stage game has a Nash equilibrium in which firms choose short-term memberships in period 0.

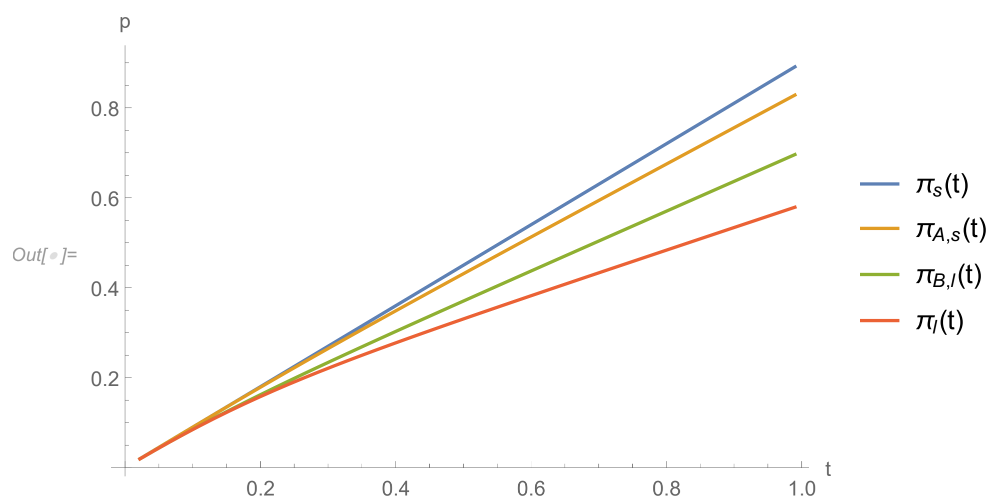
[39] The subscripts s and l stand for short- and long-term memberships.

As we mentioned before, when firms use long-term memberships, they extract a larger share of consumer surplus in period 1 with their membership fees than when they use short-term memberships. If firm A deviates from the symmetric long-term membership game and chooses short-term memberships, it has more instruments to extract consumer surplus in period 2 and can more efficiently distribute consumer surplus extraction across the two periods. Thus, in period 1, firm A has a larger market share. The same reasoning shows why firms do not have incentives to deviate from the ST-ST game.

5.3 Simulation. Here we provide a numerical example to illustrate and compare the firm's profits when using short-term and our benchmark model with long-term memberships. We use the utility function $u(p) = \frac{\alpha^{\frac{1}{\epsilon}} q^{1-\frac{1}{\epsilon}}}{1-\frac{1}{\epsilon}}$, which results in a constant elasticity demand curve (with elasticity equal to ϵ); i.e., $q(p) = \frac{\alpha}{p^{\epsilon}}$. In this case, $v(p) = \frac{1}{(\epsilon-1)} \frac{\alpha}{p^{\epsilon-1}}$. We assume the following parameters: $\alpha = 1$, $c = 2$, and $\epsilon = 2$. When $\epsilon = 2$, then $\phi(p) = \frac{p-c}{p(2c-p)}$.

Figure 5.1 shows that as the market becomes less competitive (i.e., t increases), firms offering short-term membership are more efficient at extracting surplus, (i.e., $\pi_s > \pi_{A,s} > \pi_{B,l} > \pi_l$).

FIGURE 5.1. Simulated Profits



Note: Figure 5.1 shows equilibrium profits $\{\pi_s, \pi_{A,s}, \pi_{B,l}, \pi_l\}$. Profits are shown as a function of t, with fixed value of $\delta = 0.9$.

5.4 Discussion. In some markets, firms predominantly use short-term memberships, like in the online food ordering and delivery services (e.g., *Uber Eats*, *Doordash*, and *GrubHub*) and the online grocery delivery services (e.g., *Amazon Fresh* and *Instacart*), while in other markets, long-term memberships are the norm, as in cable companies (e.g., *Spectrum* and

DirecTV). From Proposition 6 it follows that firms offer short-term memberships in equilibrium. So, why do we often see long-term memberships in reality? As we showed in Section 4, long-term memberships are useful when firms need to retain their customers in period 2. In our model, we assume that the transportation cost t (differentiation parameter) is independent of whether a user in the second period opts to stay with the same firm or chooses to buy from the rival firm. If this transportation cost is asymmetric for new and old users (e.g., users are better off staying in the same firm they purchased from in the first period), our result in Proposition 6 may not hold. Second, there could be other switching costs that would make long-term memberships more profitable. Third, it may be costly for firms to design multiple tariffs for old and new customers. Finally, there could be behavioral reasons. For example, once consumers sign up for a service, they suffer from a sunk cost fallacy and decide to keep using it, or they may be averse to paying two different membership fees.[40]

6 Extensions

In this section, we present three different extensions to our benchmark model. In Subsection 6.1, we study how the benchmark model changes when firms can offer long-term contracts in which the marginal price is fixed for both periods. We show that firms offer marginal cost-based memberships and extract surplus with the different types of fixed fees. In Subsection 6.2, we assume that firms cannot discriminate with the unit price between old and new customers (they must charge the same unit price to both customers but can still charge a fixed fee to new customers in the second period). Interestingly, marginal cost pricing is not a Nash equilibrium in this setup. Instead, firms charge a marginal price above the marginal cost. Finally, in Subsection 6.3, we compare the benchmark membership model with a standard linear pricing model with no membership fees. We show that poaching rivals' consumers is more difficult when firms use membership fees.

6.1 Long-Term Contracts (for the Unit Price). We now explore how long-term contracts impact prices in our membership (benchmark) model. Note that in our benchmark model, firms do not have commitment power; thus, the size of the two first-period markets of each firm (which depends on the difference between the first-period prices) influences prices in the second period. A natural extension of the membership model would be to allow firms to offer long-term contracts in which the marginal price is fixed for both periods.

Here, firms offer long-term contracts that include a long-term (two-period) membership fee, F_i^l, that promises to supply the goods in both periods at a fixed marginal price, p_i^l. Firms also offer the standard contract analyzed in the benchmark model: a unit price and a membership fee in the first period, (p_i, F_i); in the second period, each firm charges a single marginal price to its old customers (who did not buy the long-term contract), $p_{i,o}^2$,

[40]We thank an anonymous referee for this suggestion.

and a membership fee and marginal price to those who purchased from its rival in period 1, $\left(p_{i,n}^2, F_{i,n}^2\right)$ for $i \in \{A, B\}$.

We assume that long-term contracts are purchased by customers who mostly prefer that firm's product; that is, the set $[0, \underline{s}_A]$ and $[\underline{s}_B, 1]$ would buy long-term contracts from firms A and B, respectively. Note that consumers who buy long-term and short-term contracts from firm i are indifferent toward the two options. Consumers have homogeneous vertical taste preferences, so the model is completely deterministic for these consumers.[41]

The following proposition characterizes the whole equilibrium of the pricing game:

Proposition 7.

(i) *There is an interior equilibrium in which $p = c$, $p^l = c$, F, and F^l are defined by (A.83) and (A.84), respectively (see Appendix A);*

(ii) *In any interior symmetric equilibrium, $x^* = \frac{1}{2}$, p_o^2 is defined by (A.87), $\underline{s}_A = \frac{1}{2} - \frac{2\phi\left(p_o^2\right)+v(c)-v\left(p_o^2\right)}{4t} < s_A$, $s_A = \frac{1}{2} - \frac{v(c)-v\left(p_o^2\right)}{4t} < x^*$, $p_n^2 = c$ and $F_n^2 = \frac{v(c)-v\left(p_o^2\right)}{2t}$;*

(iii) *$F < F^l$.*

When firms use long-term contracts that guarantee a fixed marginal price for both the first and second period, they charge a (fixed) marginal price equal to the marginal cost and extract surplus through the (fixed) membership fee, F^l. Note that in this contract, the marginal prices charged in period 1 and 2 are both equal to the marginal cost, which explains why the membership fee charged in the long-term contract is higher than the membership fee for the standard membership contract; that is, $F^l > F$.

In equilibrium, $F^l - F = \delta\left(v(c) - v(p_o^2)\right)$, which suggests an inter-temporal relationship between the prices for the old customers in period 2 and the long-term and standard membership fees, F^l and F, respectively. When firms offer long-term contracts, there is a trade-off between extracting surplus from the old customers in period 2 with the marginal price and extracting surplus in period 1 with the membership fee, such that the incentive compatibility constraint is satisfied.

These results may explain the empirical regularities observed in the cable and wireless carrier markets, in which companies often offer long-term contracts: two-year contracts that specify an initial membership fee (an initial one-time fee) and a monthly price for the two years, and more flexible contracts that only specify the monthly price for the first year.

Finally, note that in Section 5, we solved the equilibrium of the three-stage game in which each firm chooses between short- or long-term memberships, and we find that short-term memberships for both firms is a Nash equilibrium. The results in this subsection and from Appendix B show that when both firms offer long-term contracts and commit to a unit

[41]Note that here the market is reduced by the share of consumers who prefer to buy the long-term contract in period 1.

price in period 2, they are worse off than when they both offer long-term memberships (see, for example, Figure B1). Following a similar strategy as in Section 5, it is not difficult to show that if firms are allowed to choose between short-term memberships and long-terms contracts with commitment (to a unit price in period 2), then short-term membership is an equilibrium. To simplify the exposition of the paper, we focus on the game in which firms choose between short- or long-term memberships without commitment.

6.2 Restricted Membership Model. If antitrust authorities were to regulate price discrimination with the unit price between old and new customers, does consumer surplus increase? Are firms harmed by this policy? How does the equilibrium change compared with our standard subscription model? In this subsection, we explore a restricted membership model in which firms are not allowed to discriminate between old and new customers with their unit price in the second period.[42]

We assume that each firm is allowed to offer a unit price and membership fee in the first period, (p_i, F_i), but in the second period, each firm offers a single marginal price to all consumers (old and new customers), p_i^2. However, a firm can also charge (subsidize) membership fees, $F_{i,n}^2$, to those who purchased from its rival in period 1 (i.e., new customers). Intuitively, firms can charge a membership fee in order to allow consumers to buy their products in both periods, but they cannot discriminate between new and old customers with the unit price.

Proposition 8. *In a symmetric equilibrium:*

(i) *in the second period, firms charge a single unit price, defined by*

$$t = \frac{3}{2}\phi\left(p^2\right)$$

and subsidize new consumers to switch with a negative membership fee, $F^2 = -\frac{\pi\left(p^2\right)}{2}$*;*

(ii) *in the first period, firms charge a marginal-cost-based 2PT with a fixed fee equal to*

$$F = t - \delta\left[\frac{\pi'\left(p^2\right)}{2} - \frac{q\left(p^2\right)\left(p^2-c\right)\left(q'\left(p^2\right)\left(p^2-c\right)\right)}{4t}\right] p_B^{2\prime}\left(x^*\right) > 0;$$

(iii) $0 < s_A = \frac{1}{2} - \frac{\pi\left(p^2\right)}{4t} < x^*$ *and* $s_B = \frac{1}{2} + \frac{\pi\left(p^2\right)}{4t} > x^*$.

We start by analyzing period 2. From Proposition 8, we conclude that when firms are not allowed to discriminate between old and new customers with their unit price (e.g., provide different monthly plans), marginal cost pricing is not an equilibrium in period 2; firms set a price above the marginal cost.

[42]That is, firms still charge a membership fee to new customers in period 2 but offer the same marginal price to both old and new customers in period 2.

In a symmetric equilibrium, $x^* = \frac{1}{2}$, each firm offers subsidies to the new customers, proportional to its monopoly profits. Since firms know consumers' optimal demand, they set subsidies (negative fixed fees) that still allow them to extract positive surplus from the switchers.[43] Note that if firm i sets a fixed fee equal to 0, it is optimal for its rival (firm $j \neq i$) to set a negative fixed fee; firm i will not poach consumers from j's turf in equilibrium, while firm j poaches a positive share from i's turf and gets a positive net revenue from them. If firm i increases its price, firm j has no incentive to increase its price; by keeping its price constant (or lower), it increases the share of switchers, increasing the net revenue. This means that firms cannot extract surplus through a fixed fee from new customers. Instead, they need to offer subsidies proportional to the customers' demand and now extract surplus through the unit price.[44] In period 1, firms offer cost-based membership fees. A unit price equal to the marginal cost and a positive membership fee, which follows from the symmetry of the two firms.

These results may explain the empirical regularities observed in the wireless carrier market. Often, firms charge an initial membership fee to consumers (e.g., for the cell phone) and a marginal price for the monthly plan. Usually, these companies offer the same monthly plan to current and new customers, although they discriminate with their subscription fee.[45] For the US market, most of the big carriers offer subsidies to trade in the phone from rival firms, holding the price for the monthly plans fixed for new and current customers.[46] Here we explore how the unit prices and membership fees change when firms are restricted to offering the same unit price (or monthly price) to new and current customers.

6.3 Comparison with the Linear Pricing Game. In this subsection, we compare our membership model with a standard linear pricing (LP) model with no membership fees. We show that the marginal prices offered to old customers in the LP game are lower than the marginal prices offered in the benchmark model. Moreover, we show that the share of switchers in period 2 is smaller in the subscription model compared with the LP model; that is, poaching a rival's customers is more difficult when firms use membership fees.

[43] This fact explains, why quasi-best response functions with respect to the marginal price are decreasing.

[44] The result in Proposition 8(i) contrasts with related results found in the literature on competitive price discrimination. Particularly, Tamayo and Tan [21] show that in a standard static framework with horizontally differentiated consumers with homogeneous taste preferences and asymmetric firms, marginal-cost-based two-part tariff is a unique equilibrium. The difference here is that now each firm sets a price for both markets; that is, both firm A's and firm B's turf. Thus, marginal-cost-based 2PT is no longer an equilibrium strategy; otherwise, in any symmetric equilibrium, it would not be possible to poach consumers.

[45] As far as we are aware, during 2016-2017, only *Sprint* offered different monthly plans that were conditional on whether the client was a new or old customer.

[46] *Verizon* offers up to \$650 for an installment plan balance less trade-in value (or an up-to-\$350 prepaid card for early termination fees less trade-in value), to customers who switch to *Verizon*. *T-Mobile*, *Sprint*, and *AT&T* offer similar plans.

The membership fee is an extra pricing instrument that allows firms to extract surplus more efficiently. We may expect that this extra tool will harm consumers compared with the case in which firms use only LP.[47]

Let $p^2_{2PT,o}$ be the price offered to old customers in equilibrium in our membership model presented above (Propositions 1 and 2), and let $p^2_{LP,n}$ and $p^2_{LP,o}$ be the marginal price offered to new and old customers when firms use linear pricing, respectively, in period 2. The following corollary compares these marginal prices and the share of switchers for both cases:

Corollary 4. *In equilibrium,*

(i) $p^2_{LP,n} < p^2_{LP,o} < p^2_{2PT,o}$;
(ii) $s^A_{LP} < s^A_{2PT}$.

Therefore, the membership fee allows firms to extract surplus more efficiently from new customers, which also allows the rival firm to set a higher marginal price for its old customers. Membership fees not only help firms extract surplus from consumers on their rival's turf but also, given that firms have upward slope reaction curves, allows them to increase the marginal price charge to old customers on their own turf, compared with the price offered in the LP game. Similarly, the surplus extracted from new customers is also greater in the membership model. Thus, in equilibrium, both firms extract more consumer surplus on both turfs, reducing consumers' welfare. Note that poaching a rival's customers is more difficult when firms use membership fees; fixed fees may create sunk costs (switching costs), which allow them to extract a higher share of consumers' surplus, diminishing the proportion of switchers.

7 Conclusions

In this paper, we study competition and consumer behavior in membership (subscription) markets. We consider a competitive two-period membership (subscription) market, in which two symmetric firms charge a membership fee that allows consumers to buy products or services at a given unit price in both periods. Should firms choose long- or short-term memberships? Our framework shows that, in equilibrium, firms choose short-term memberships. When firms offer short-term memberships, they have more instruments to extract consumer surplus in period 2, which allows them to more efficiently distribute consumer surplus extraction across the two periods. In contrast, with long-term memberships, firms need to extract a larger portion of their profits in period 1.

In our benchmark model, firms use two-period membership fees and charge a unit (marginal) price for their products/services on each period. Old customers don't need to pay the

[47]Note that a model in which firms use linear pricing will be close to the model proposed by Fudenberg and Tirole [11], but instead of buying a unit good, consumers have elastic demands. In Appendix D, we provide the details of this model.

membership fee again in period 2 if they buy from the same firm they bought from in period 1, but they need to pay a price for each unit they buy in both periods. In the second period, firms discriminate based on prior purchase behavior and charge a single marginal price to their old customers and a subscription fee and a differentiated marginal price to their new customers (those who purchased from the rival in period 1). In equilibrium, firms charge higher unit prices to their old customers and charge cost-based membership fees to their new customers. In period 1, firms charge cost-based membership fees in equilibrium.

However, with short-term membership (i.e., consumers must renew their memberships to buy the products in the second period, so the membership fees are paid in both periods), we show that in equilibrium, firms offer marginal-cost pricing in both periods and extract surplus through membership fees from both new and old customers. Therefore, they do not discriminate with their marginal price between old and new customers; instead, they offer differentiated membership fees. Overall, the number of consumers poached is smaller with long-term memberships but the equilibrium profits are higher when firms offer short-term memberships. Given that poaching erodes consumers' welfare and firms are better off extracting surplus with short-term memberships, consumers are better off with long-term memberships.

To understand what tariff firms choose in equilibrium, we study the asymmetric game in which one firm offers long-term and the other firm offers short-term memberships. We show that when firms use long-term memberships, they extract a larger share of consumer surplus in period 1 with their membership fee than when firms use short-term memberships. Next, we endogenize the pricing decision of firms between long- and short-term memberships. We consider a three-period game, in which in period 0, each firm chooses between long- or short-term memberships and in period 1 and 2, the firms offer long- or short-term memberships as described above. We show that firms have incentives to deviate from the symmetric long-term membership game and instead offer short-term memberships. Moreover, firms do not have incentives to deviate from the symmetric short-term membership game. Thus, short-term membership is a Nash equilibrium.

We extend our analysis further by assuming that firms offer long-term contracts with their unit price. In equilibrium, firms set the long-term marginal price equal to the marginal cost and extract surplus through the long-term contract membership fee, which is higher than the standard membership fee. When firms offer long-term memberships with long-term contracts with the unit price, captive demand of old customers in period 2 decreases (by those who accepted the long-term contract in period 1), decreasing the overall profits. Second, we consider a model in which firms are not allowed to price discriminate based on purchase history (i.e., between old and new customers) with their unit price. However, they are allowed to charge (subsidize) membership fees. We show that marginal-cost pricing is not an equilibrium in period 2 and instead firms offer a subsidy proportional to the monopoly profit function with the fixed fee (i.e., negative fixed fees). Finally, we compare our benchmark membership model with a standard linear pricing model with no membership fees.

TABLE 1. Examples of Industries with Memberships

		Consumers Monthly (millions)	Firms	Pricing Strategy	Examples	Discrimination New Customers: Membership Fee	Discrimination New Customers: Unit Price	Discrimination High- and Low-Type: Membership Fee	Discrimination High- and Low-Type: Unit Price
On-demand economy is attracting more than 22.4 million consumers annually and $57.6 billion in spending.	Online marketplaces	90 NA	*Amazon* *EBay*	Linear pricing & short-term	Amazon Prime: $99 a year + unit price of the	30 day Prime Trial	No	No	No
	Transportation	40	*Uber* *Lyft*	Linear pricing & short-term membership	*Uber* Plus: $49 per month and $3 flat rebate for *Uber* pool ride and $8 for UberX	Discount first 2 weeks	No	No	Yes
	Food-grocery delivery	28% US HH	*Instacart* *Amazon Fresh*	Short-term membership	$149 per year $14.99 per month	Free trial for 14 days Free trial for 30 days	No No	No No	No No

Note: The number of Prime members corresponding to the month of September 2017 is from: Consumer Intelligence Research Partners. Number of Amazon Prime members in the United States as of September 2017 (in millions): https://www.statista.com/statistics/546894/number-of-amazon-prime-paying-members/ (accessed November 13, 2017). The number of *Uber* riders is around 40 millions per month and is from: Kia Kokalitcheva (October 20, 2016). "Uber Now Has 40 Million Monthly Riders Worldwide;" Fortune magazine. Nielsen estimated that 28% of the US households were shopping groceries online (Nielsen Digital Shopper Fundamentals and Category Shopping Fundamentals survey, 2016).

	Firms	Subscribers (millions)	Pricing Strategy	Examples	Discrimination New Customers: Membership Fee	Discrimination New Customers: Unit Price	Discrimination High- and Low-Type: Membership Fee	Discrimination High- and Low-Type: Unit Price
Wireless Carriers	*Sprint*	58.741	Long-term (lifetime) membership	Membership fee + monthly plan	Yes	Yes (different monthly plans)	No information	Yes (lower monthly rates per line for high types)
	Verizon	146.013	Long-term (lifetime) membership	Membership fee + monthly plan	Yes (trade-in offers)	No	Yes (lower price on accessories)	Yes (lower monthly rates per line for high types)
	AT&T	134.218	Long-term (lifetime) membership	Membership fee + monthly plan	Yes (trade-in offers)	No	Yes (lower price on phones & accessories)	Yes (lower monthly rates per line for high types)
	T-Mobile	72.597	Long-term (lifetime) membership	Membership fee + monthly plan	Yes (trade-in offers)	No	Yes (lower price on phones & accessories)	Yes (lower monthly rates per line for high types)

Note: The number of subscribers is for the first quarter of 2017; data from Mike Dano, Fierce Wireless (May 8, 2017). Sprint offers monthly plans to new customers conditional to have a current port with AT&T, Verizon or TMobile. The biggest carriers (e.g., Verizon, T-Mobile and AT&T) have trade-in offers, which include cash back, cover of switching fees, among others. Moreover, the main carriers (AT&T, Verizon, TMobile and Sprint) offer business plans (with different monthly rates and membership fees)

	Firms	Subscribers (millions)	Pricing Strategy	Examples	Discrimination New Customers: Membership Fee	Discrimination New Customers: Unit Price	Discrimination High- and Low Type: Membership Fee	Discrimination High- and Low Type: Unit Price
Cable Companies	*Comcast/Xfinity*	22,549	Long-term (lifetime) membership	Membership fee + monthly plan	Yes (activation, installation & equipment fees)	Yes (monthly plans to new customers only)	N/A	Yes (differentiated monthly rates for business)
	Charter/Spectrum	17,147	Long-term (lifetime) membership	Membership fee + monthly plan	Yes (activation, installation & equipment fees)	Yes (monthly plans to new customers only)	N/A	Yes (differentiated monthly rates for business)
	DirecTV	21,012	Long-term (lifetime) membership	Membership fee + monthly plan	Yes (activation, installation & equipment fees)	Yes (monthly plans to new customers only)	Yes (lower price on phones & accessories)	Yes (differentiated monthly rates for business)

Note: The number of subscribers is for the first quarter of 2017; data from Leichtman Research Group, Press Release (May 18, 2017). Comcast pricing offers distinguish by old and new customers e.g. "New residential customers only" and "new Comcast business customers only" for the plans specific to business. It also requires subscriptions to the services besides the equipment, installation, taxes, and other applicable charges extra (life-time membership). In the recent merge between Time Warner Cable and Spectrum, the last offer limited monthly plan offers to customers who were not subscribed to applicable services within the previous 30 days. Directv offers packages that require 24-month agreement, with a discount in the monthly rate for the first year only, and initial (lifetime) subscription fees.

Figure 7.1. *DirecTV* Plans Offered

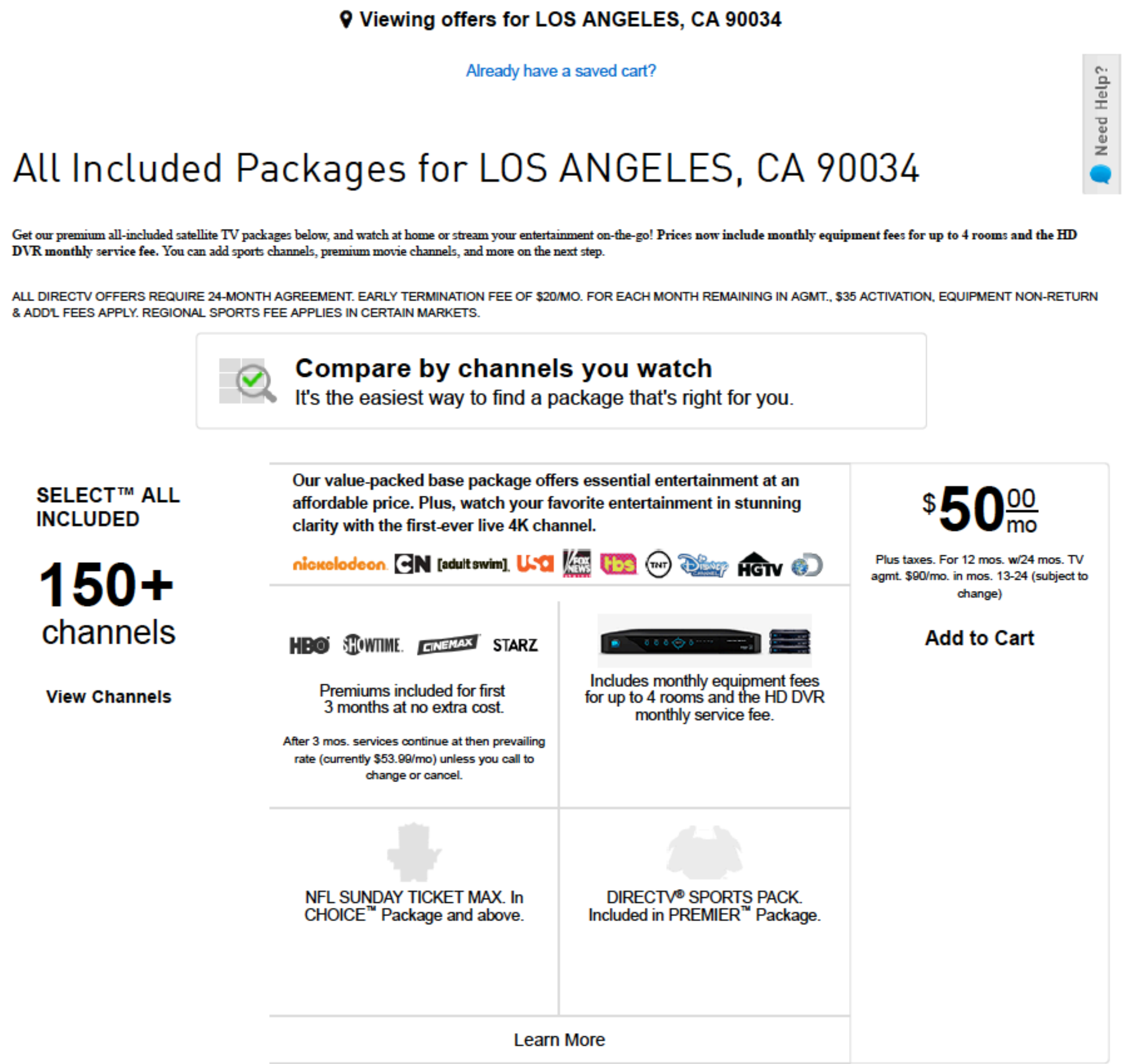
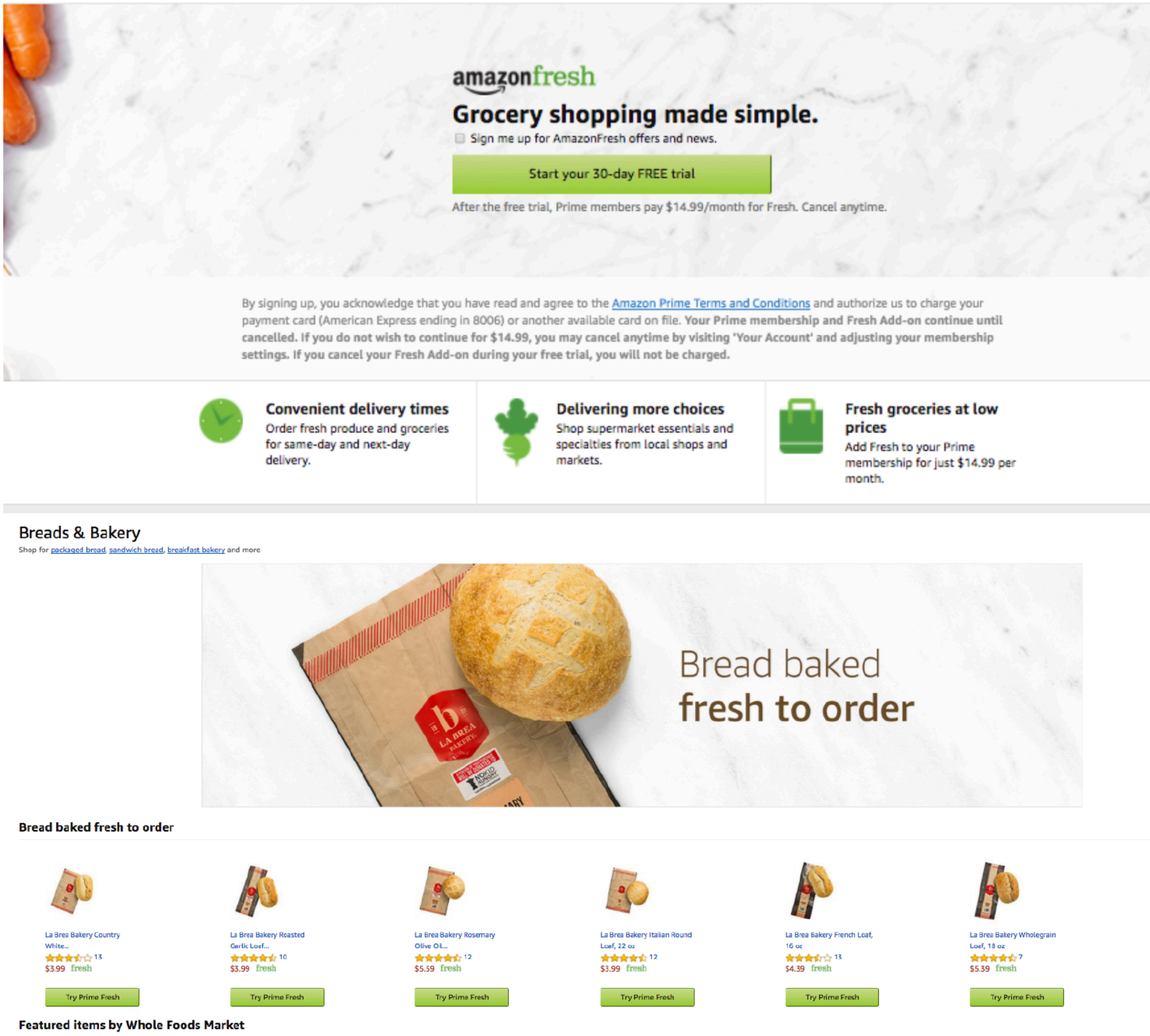


FIGURE 7.2. *Amazon Fresh* Membership and Unit Prices



Appendix A: Proofs

Proof of Proposition 1.

(i) The first-order condition of firm A with respect to $p_{A,o}^2$ is

$$\pi'\left(p_{A,o}^{2*}\right)\left\{2t\cdot s_A\left(p_{A,o}^{2*},p_{B,n}^2,F_{B,n}^2\right)-\phi\left(p_{A,o}^{2*}\right)\right\}=0. \tag{A.1}$$

From (A2) it follows that $2t\cdot s_A\left(p_{A,o}^{2*},p_{B,n}^2,F_{B,n}^2\right)-\phi\left(p_{A,o}^{2*}\right)$ is strictly decreasing with respect to $p_{A,o}^2$ for any $p_{A,o}^2>c$. Note that if there exists a $\tilde{p}_{A,o}^2\left(p_{B,n}^2,F_{B,n}^2\right)\in\mathcal{P}$ such that $s\left(\tilde{p}_{A,o}^2,p_{B,n}^2,F_{B,n}^2\right)=0$, then for any $p_{A,o}^2\geq\tilde{p}_{A,o}^2$, the profit is zero. Note that (A.1) is positive for $p_{A,o}^2<p_{A,o}^{2*}$ and it is negative for any $p_{A,o}^2>p_{A,o}^{2*}$. Thus (2.1) is single-peaked in p_A^2 and reaches a unique maximum at $p_{A,o}^2=p_{A,o}^{2*}$, implicitly defined by (A.1).

We now solve the two-variable optimization problem of firm B on A's turf sequentially. First, we show that for any $p_{B,n}^2\in\mathcal{P}$, firm B chooses $F_{B,n}^2$ to maximize its profits. The first-order condition with respect to $F_{B,n}^2$ yields

$$\left(x^*-\frac{1}{2}-\frac{v\left(p_{A,o}^2\right)-v\left(p_{B,n}^2\right)+F_{B,n}^2}{2t}\right)-\frac{1}{2t}\left(\pi\left(p_{B,n}^2\right)+F_{B,n}^2\right)=0. \tag{A.2}$$

The profit is quadratic and strictly concave in $F_{B,n}^2$, and the unique solution is given by

$$F_{B,n}^{2*}=\frac{2tx^*-t-v\left(p_{A,o}^2\right)+v\left(p_{B,n}^2\right)-\pi\left(p_{B,n}^2\right)}{2}. \tag{A.3}$$

Next, firm B chooses $p_{B,n}^2$ to maximize its maximum profits (we substitute $F_{B,n}^{2*}\left(p_{B,n}^2\right)$ in 2.2):

$$\left(x^*-s_A\left(p_{A,o}^2,p_{B,n}^2,F_{B,n}^{2*}\left(p_{B,n}^2\right)\right)\right)\left(\pi\left(p_{B,n}^2\right)+F_{B,n}^{2*}\left(p_{B,n}^2\right)\right). \tag{A.4}$$

The derivative of (A.4) with respect to $p_{B,n}^2$, after using the envelope theorem and (A.2), is

$$\left(x^*-s_A\left(p_{A,o}^2,p_{B,n}^2,F_{B,n}^{2*}\left(p_{B,n}^2\right)\right)\right)q'\left(p_{B,n}^2\right)\left(p_{B,n}^2-c\right)=0. \tag{A.5}$$

Given x^* and $p_{A,o}^2\in\mathcal{P}$, $x^*-s_A\left(p_{A,o}^2,p_{B,n}^2,F_{B,n}^{2*}\left(p_{B,n}^2\right)\right)$, is strictly decreasing with respect to $p_{B,n}^2$. Thus, if there exists $\underline{x}\in[0,x^*]$ and $\tilde{p}_{B,n}^2\left(p_{A,o}^2\right)\in\mathcal{P}$, such that $\underline{x}-s_A\left(p_{A,o}^2,p_{B,n}^2,F_{B,n}^{2*}\left(p_{B,n}^2\right)\right)=0$, then for any $p_{B,n}^2\geq\tilde{p}_{B,n}^2$, the profit is zero. Then for any $p_{A,o}^2\in\mathcal{P}$, note that (A.5) is positive for $p_{B,n}^2<c$ and it is negative for any $p_{B,n}^2\in\left(c,\tilde{p}_{B,n}^2\left(p_{A,o}^2,\underline{x}\right)\right)$ and $x^*>\underline{x}$. Thus, (A.4) is single-peaked in $p_{B,n}^2$ and reaches a unique maximum at $p_{B,n}^2=c$.

Note that if $t<v\left(c\right)$, as per previous paragraphs and the intermediate value theorem, there exists $\underline{\mathrm{x}}<\frac{1}{2}$ such that for $x^*\geq\underline{\mathrm{x}}$, there is a unique interior equilibrium in which

$$p_{B,n}^{2*} = c, \quad F_{B,n}^{2*} = \frac{2tx^* - t - v\left(p_{A,o}^{2^*}\right) + v\left(c\right)}{2}$$

and $p_{A,o}^{2*}$ is defined by[48]

$$t - v\left(c\right) + 2tx^* = \psi\left(p_{A,o}^{2*}\right),$$

where $\psi\left(p\right) \equiv 2\phi\left(p\right) - v\left(p\right)$ and $\phi\left(p\right) \equiv \frac{q(p)\pi(p)}{\pi'(p)}$. Note that $\psi'\left(\cdot\right) > 0$, so $p_{A,o}^{2*}$ is uniquely defined by $\psi^{-1}\left(t - v\left(c\right) + 2tx^*\right)$.

The problem on B's turf is symmetric; that is, there exists $\overline{x} > \frac{1}{2}$ such that for $x^* \leq \overline{x}$, there is a unique interior equilibrium on B's turf.

(ii) Follows directly from the fact that $t < v\left(c\right)$ and the definition of $p_{A,o}^{2*}$ and $F_{B,n}^{2*}$.

Proof of Corollary 1.

(i) From the first-order condition of firm A on its own turf,

$$2tx^* + t - v(c) - 2\phi\left(p_{A,o}^{2}\right) + v\left(p_{A,o}^{2}\right) = 0.$$

Thus,

$$\frac{\partial p_{A,o}^{2}}{\partial x^*} = \frac{2t}{2\phi'\left(p_{A,o}^{2}\right) + q\left(p_{A,o}^{2}\right)} > 0.$$

Similarly, for firm B on A's turf,

$$\frac{\partial F_{B,n}^{2}}{\partial x^*} = t + \frac{q\left(p_{A,o}^{2}\right)}{2}\frac{\partial p_{A,o}^{2}}{\partial x^*} = t + \frac{t \cdot q\left(p_{A,o}^{2}\right)}{2\phi'\left(p_{A,o}^{2}\right) + q\left(p_{A,o}^{2}\right)}.$$

(ii) Similarly, note that

$$\frac{\partial p_{B,o}^{2}}{\partial x^*} = \frac{-2t}{2\phi'\left(p_{B,o}^{2}\right) + q\left(p_{B,o}^{2}\right)} \quad \text{and} \quad \frac{\partial F_{A,n}^{2}}{\partial x^*} = -t - \frac{t \cdot q\left(p_{B,o}^{2}\right)}{2\phi'\left(p_{B,o}^{2}\right) + q\left(p_{B,o}^{2}\right)}.$$

Proof of Proposition 2.

(i) From the results in Proposition 1, in the symmetric equilibrium $x^* = 1/2$,

$$p_{n}^{2*} = c, \quad F_{n}^{2*} = \frac{v\left(c\right) - v\left(p_{o}^{2^*}\right)}{2}$$

and p_{o}^{2*} is defined by,

$$2t - v\left(c\right) = \psi\left(p_{o}^{2*}\right),$$

where $\psi\left(p\right) \equiv 2\phi\left(p\right) - v\left(p\right)$ and $\phi\left(p\right) \equiv \frac{q(p)\pi(p)}{\pi'(p)}$.

[48]Note that $p_{A,o}^{2*} > c$ given that $0 < t < v\left(c\right)$, which implies that $\psi\left(p_{A,o}^{2*}\right) > \psi\left(c\right)$.

(ii) The problem of the first period of firm A is

$$\text{(A.6)} \qquad \max_{p_A, F_A} \quad x^*(\alpha)\left(\pi(p_A)+F_A\right)+\delta s_A\left(p^2_{A,o}, p^2_{B,n}, F^2_{B,n}\right)\pi\left(p^2_{A,o}\right)$$

$$+\delta\left[s_B\left(p^2_{B,o}, p^2_{A,n}, F^2_{A,n}\right)-x^*(\alpha)\right]\left(F^2_{A,n}+\pi\left(p^2_{A,n}\right)\right).$$

Using the results in Proposition 1, the first-order condition after using the envelope theorem with respect to p_A is

$$\text{(A.7)} \quad \frac{\partial x^*(\alpha)}{\partial p_A}\left(\pi(p_A)+F_A\right)+x^*(\alpha)\pi'(p_A)+\delta\left\{\frac{\partial s_A\left(p^2_{A,o}, c, F^2_{B,n}\right)}{\partial F^2_{B,n}}\pi\left(p^2_{A,o}\right)\frac{\partial F^2_{B,n}}{\partial x^*}\right\}\frac{\partial x^*}{\partial p_A}$$

$$+\delta\left[s_B\left(p^2_{B,o}, c, F^2_{A,n}\right)-x^*(\alpha)\right]\left(\frac{\partial F^2_{A,n}}{\partial x^*}\right)\frac{\partial x^*}{\partial p_A}$$

$$+\delta\left[\frac{\partial s_B\left(p^2_{B,o}, c, F^2_{A,n}\right)}{\partial p^2_{B,o}}\frac{\partial p^2_{B,o}}{\partial x^*}+\frac{\partial s_B\left(p^2_{B,o}, c, F^2_{A,n}\right)}{\partial F^2_{A,n}}\frac{\partial F^2_{A,n}}{\partial x^*}-1\right]F^2_{A,n}\frac{\partial x^*}{\partial p_A}=0$$

and with respect to F_A,

$$\text{(A.8)} \quad \left(\frac{\partial x^*(\alpha)}{\partial F_A}\right)\left(\pi(p_A)+F_A\right)+x^*(\alpha)+\delta\left\{\frac{\partial s_A\left(p^2_{A,o}, c, F^2_{B,n}\right)}{\partial F^2_{B,n}}\pi\left(p^2_{A,o}\right)\frac{\partial F^2_{B,n}}{\partial x^*}\right\}\left(\frac{\partial x^*(\alpha)}{\partial F_A}\right)$$

$$+\delta\left[s_B\left(p^2_{B,o}, c, F^2_{A,n}\right)-x^*(\alpha)\right]\left(\frac{\partial F^2_{A,n}}{\partial x^*}\right)\left(\frac{\partial x^*(\alpha)}{\partial F_A}\right)$$

$$+\delta\left[\frac{\partial s_B\left(p^2_{B,o}, c, F^2_{A,n}\right)}{\partial p^2_{B,o}}\frac{\partial p^2_{B,o}}{\partial x^*}+\frac{\partial s_B\left(p^2_{B,o}, c, F^2_{A,n}\right)}{\partial F^2_{A,n}}\frac{\partial F^2_{A,n}}{\partial x^*}-1\right]F^2_{A,n}\left(\frac{\partial x^*(\alpha)}{\partial F_A}\right)=0.$$

Note that from (A.8) and (A.7), it follows that

$$\text{(A.9)} \qquad x^*(\alpha)\left(\frac{\partial x^*(\alpha)}{\partial F_A}\right)^{-1}-x^*(\alpha)\pi'(p_A)\left(\frac{\partial x^*(\alpha)}{\partial p_A}\right)^{-1}=0.$$

From (2.3), we know that

$$\text{(A.10)} \qquad \frac{\partial x^*}{\partial p_A}=\frac{-q(p_A)}{2t(1-\delta)-2\delta F^{2\prime}_{A,n}(x^*)}$$

and

(A.11)
$$\frac{\partial x^*}{\partial F_A} = \frac{-1}{2t(1-\delta) - 2\delta F^{2\prime}_{A,n}(x^*)}.$$

By noting that the denominators of (A.10) and (A.11) are different from zero, and using (A.10) and (A.11) in (A.9),

(A.12)
$$\frac{x^*(\alpha)}{q(p_A)}\left(\frac{\partial x^*(\alpha)}{\partial F_A}\right)^{-1} q'(p_A)(p_A - c) = 0.$$

Given $F_B^* \geq 0$ and $p_B \in \mathcal{P}$, $x^*(\alpha)$, is strictly decreasing with respect to p_A. Thus, if there exists $\tilde{p}_A(\tilde{\alpha}) \in \mathcal{P}$ such that $x^*(\alpha) = 0$, then for any $p_A \geq \tilde{p}_A(\tilde{\alpha})$, the profit is zero. Then, for any $F_B^* \geq 0$ and $p_B \in \mathcal{P}$, note that (A.12) is positive for $p_A < c$ and negative for any $p_A \in (c, \tilde{p}_A(\tilde{\alpha}))$. Thus, (A.6) is single-peaked in p_A and reaches a maximum at $p_A = c$. Thus, we conclude that any equilibrium involves marginal-cost pricing.

In any symmetric equilibrium, $x^* = \frac{1}{2}$, $p^* = c$, and

(A.13)
$$F^* = t(1-\delta) - \delta\frac{\partial F_n^2}{\partial x^*} - \delta\left\{\frac{1}{2t}\pi\left(p_o^2\right)\frac{\partial F_n^2}{\partial x^*}\right\}$$
$$-\delta\left[\frac{v(c) - v(p_o^2)}{4t}\right]\left(\frac{\partial F_n^2}{\partial x^*}\right) - \delta\left[\frac{q(p_o^2)}{2t}\frac{\partial p_o^2}{\partial x^*} + \frac{1}{2t}\frac{\partial F_n^2}{\partial x^*} - 1\right]F_n^{2*}.$$

Thus from Corollary 1, and using (A.10) and (A.11) in (A.13) we have

$$F^* = t + \delta\frac{t \cdot q(p_o^{2*})}{2\phi'(p_o^{2*}) + q(p_o^{2*})} + \delta\frac{\phi'(p_o^{2*}) + q(p_o^{2*})}{2\phi'(p_o^{2*}) + q(p_o^{2*})}\left(v(c) - v\left(p_o^{2*}\right) - \pi\left(p_o^{2*}\right)\right);$$

From (iii), $F^* > F_n^2$, it then follows that $F^* > 0$.

(iii) Note that in a symmetric equilibrium,

$$t = \phi\left(p_o^{2*}\right) + \frac{v(c) - v(p_o^{2*})}{2};$$

thus, it follows that $F^* > F_n^{2*}$ since $\phi(p) > \pi(p)$ for any $p \in \mathcal{P}$.

(iv) Follows from the fact that $p_o^{2*} > c$.

Proof of Proposition 3. In period 2, the problem of firm A on its own turf is now:

(A.14)
$$\max_{p^2_{A,o}, F^2_{A,o}} \quad s_A\left(p^2_{A,o}, F^2_{A,o}, p^2_{B,n}, F^2_{B,n}\right)\left(\pi\left(p^2_{A,o}\right) + F^2_{A,o}\right),$$

where $s_A\left(p^2_{A,o},F^2_{A,o},p^2_{B,n},F^2_{B,n}\right)\equiv\frac{1}{2}+\frac{v\left(p^2_{A,o}\right)-v\left(p^2_{B,n}\right)-F^2_{A,o}+F^2_{B,n}}{2t}$. Given the cut-off, x^*, the problem of firm B on A's turf is

$$\text{(A.15)}\qquad \max_{p^2_{B,n},F^2_{B,n}}\left(x^*-s_A\left(p^2_{A,o},F^2_{A,o},p^2_{B,n},F^2_{B,n}\right)\right)\left(\pi\left(p^2_{B,n}\right)+F^2_{B,n}\right).$$

Note that on each turf, firms are playing an asymmetric 2PT game similar to the model with homogeneous taste parameters studied by [21]. Here, we adapt Proposition 1 in [21] to show that there exist $\underline{x}$ and $\overline{x}$ such that marginal cost-based 2PT is a unique equilibrium in period 2. We solve the two-variable optimization problem of firm A sequentially. First, we show that for any $p^2_{A,o}\in\mathcal{P}$, firm A chooses $F^2_{A,o}$ to maximize its profits. The first-order condition with respect to $F^2_{A,o}$ yields

$$\text{(A.16)}\qquad s_A\left(p^2_{A,o},F^2_{A,o},p^2_{B,n},F^2_{B,n}\right)-\frac{1}{2t}\left[\pi\left(p^2_{A,o}\right)+F^2_{A,o}\right]=0.$$

The profit is quadratic and strictly concave in $F^2_{A,o}$, and the unique solution is given by

$$\text{(A.17)}\qquad 2F^{2*}_{A,o}=t+v\left(p^2_{A,o}\right)-v\left(p^2_{B,n}\right)+F^2_{B,n}-\pi\left(p^2_{A,o}\right).$$

Next, firm A chooses $p^2_{A,o}$ to maximize its profits (substitute $F^{2*}_{A,o}\left(p^2_{A,o}\right)$ in A.14):

$$\text{(A.18)}\qquad s_A\left(p^2_{A,o},F^{*2}_{A,o}\left(p^2_{A,o}\right),p^2_{B,n},F^2_{B,n}\right)\left[\pi\left(p^2_{A,o}\right)+F^{*2}_{A,o}\left(p^2_{A,o}\right)\right].$$

The derivative of (A.18) with respect to $p^2_{A,o}$, after using the envelope theorem, is

$$\text{(A.19)}\qquad q'\left(p^2_{A,o}\right)\left(p^2_{A,o}-c\right)s_A\left(p^2_{A,o},F^{*2}_{A,o}\left(p^2_{A,o}\right),p^2_{B,n},F^2_{B,n}\right)=0.$$

Given $p^2_{B,n}\in\mathcal{P}$ and $F^2_{B,n}\geq 0$,

$$s_A\left(p^2_{A,o},F^{*2}_{A,o}\left(p^2_{A,o}\right),p^2_{B,n},F^2_{B,n}\right)={}^1\!/_{4t}\left\{t+v\left(p^2_{A,o}\right)-v\left(p^2_{B,n}\right)+F^2_{B,n}+\pi\left(p^2_{A,o}\right)\right\},$$

is strictly decreasing with respect to $p^2_{A,o}$ for any $p^2_{A,o}>c$.

Thus, if there exists a $\tilde{p}^2_{A,o}\left(p^2_{B,n},F^2_{B,n}\right)\in\mathcal{P}$ such that $s_A\left(\tilde{p}^2_{A,o},F^{*2}_{A,o}\left(\tilde{p}^2_{A,o}\right),p^2_{B,n},F^2_{B,n}\right)=0$, then for any $p^2_{A,o}\geq\tilde{p}^2_{A,o}$, the profit is zero. Then, for any $p^2_{B,n}\in\mathcal{P}$ and $F^2_{B,n}\geq 0$, note that (A.19) is positive for $p^2_{A,o}<c$ and negative for any $p^2_{A,o}\in\left(c_i,\tilde{p}^2_{A,o}\left(p^2_{B,n},F^2_{B,n}\right)\right)$. Thus, (A.18) is single-peaked in $p^2_{A,o}$ and reaches a unique maximum at $p^2_{A,o}=c$. Analogously, the profit function for firm B on A's turf as a function of $p^2_{B,n}$ is single-peaked in $p^2_{B,n}$ and reaches a unique maximum at $p^2_{B,n}=c$ for $x\geq\underline{x}$. Thus, the equilibrium is unique.

Lets now analyze the problem of Firm A in period 1. The problem in the first-period of firm A is

$$\max_{p_A, F_A} \quad x^*(\alpha)(\pi(p_A) + F_A) + \delta s_A\left(p_{A,o}^2, F_{A,o}^2, p_{B,n}^2, F_{B,n}^2\right)\left(\pi\left(p_{A,o}^2\right) + F_{A,o}^2\right) \tag{A.20}$$

$$+\delta\left[s_B\left(p_{B,o}^2, F_{B,o}^2, p_{A,n}^2, F_{A,n}^2\right) - x^*(\alpha)\right]\left(F_{A,n}^2 + \pi\left(p_{A,n}^2\right)\right).$$

We follow a similar strategy as in Proposition 2 to show that any equilibrium involves marginal-cost-based 2PT. Note that the problem is concave in F_A. Moreover, from the first-order conditions with respect to p_A and F_A, it follows that

$$x^*(\alpha)\, q'(p_A)(p_A - c) = 0. \tag{A.21}$$

Given $F_B \geq 0$ and $p_B \in \mathcal{P}$, $x^*(\alpha)$, is strictly decreasing with respect to p_A for any $p_A > c$. Thus, if there exists $\tilde{p}_A(\tilde{\alpha}) \in \mathcal{P}$ such that $x^*(\tilde{\alpha}) = 0$, then for any $p_A \geq \tilde{p}_A(\tilde{\alpha})$, the profit is zero. Then, for any $F_B \geq 0$ and $p_B \in \mathcal{P}$, note that (A.21) is positive for $p_A < c$ and negative for any $p_A \in (c, \tilde{p}_A(\tilde{\alpha}))$. Thus, (A.20) is single-peaked in p_A and reaches a maximum at $p_A = c$.

Finally, we derive F_A^*. Note that from the first-order conditions with respect to F_A after using the envelope theorem, and that in any symmetric equilibrium,

$$\frac{\partial F_{A,o}^2}{\partial x^*} = \frac{2t}{3}, \quad \frac{\partial F_{B,n}^2}{\partial x^*} = \frac{4t}{3}, \quad \frac{\partial x^*}{\partial F_A} = \frac{-3}{2(3+\delta)t},$$

$$F_o^2 = \frac{2t}{3}, \quad \text{and} \quad F_n^2 = \frac{t}{3},$$

then we have

$$F_A^* = \frac{(3+\delta)}{3}t.$$

Lemma A1. *Given (A2), for any $p > c$*

(i) $\phi'(p) > q(p)$;

(ii) $\phi(p) > v(c) - v(p)$.

Proof of Lemma A1. (i) Since $\phi(p) \equiv \frac{q(p)\pi(p)}{\pi'(p)}$,

$$\phi'(p) - \frac{q(p)^2}{\pi'(p)} = \frac{\pi'(p)\, q(p)\, \pi'(p) + \pi(p)\, q'(p)\, \pi'(p) - \pi(p)\, q(p)\, \pi''(p)}{\pi'(p)^2} - \frac{q(p)^2}{\pi'(p)}. \tag{A.22}$$

From $\pi'(p) = q(p) + q'(p)(p - c)$ and $\pi''(p) = 2q'(p) + q''(p)(p - c)$ in (A.22),

$$\phi'(p) - \frac{q(p)^2}{\pi'(p)} = \frac{q(p)\, q'(p)^2}{\pi'(p)^2}(p-c)^2 \underbrace{\left\{2 - \frac{q''(p)\, q(p)}{q'(p)^2}\right\}}_{>0, \text{ by (A2)}}. \tag{A.23}$$

From (A.23), $\phi'(p) - \frac{q(p)^2}{\pi'(p)} > 0$. Thus, for any $p > c$

$$\phi'(p) > \frac{q(p)^2}{q'(p)(p-c) + q(p)} \geq q(p).$$

(ii) First notice that

$$[\phi(p) - v(c) + v(p)]\,\big|_{p=c} = 0,$$

and

$$\begin{aligned} [\phi(p) - v(c) + v(p)]' &= \frac{(p-c)\, q(p)\left((p-c)\, q'(p)^2 - q(p)\left((p-c)\, q''(p) + q'(p)\right)\right)}{\pi'(p)^2} \\ &= \frac{(p-c)\, q(p)\, M(p)}{\pi'(p)^2}, \end{aligned} \tag{A.24}$$

where $M(p) \equiv (p-c)\, q'(p)^2 - q(p)\left((p-c)\, q''(p) + q'(p)\right)$. The term $M(p)$ can be rewritten as

$$M(p) = q'(p)^2 (p-c) \underbrace{\left[1 - \frac{q(p)\, q''(p)}{q'(p)^2}\right]}_{>-1, \text{ by (A2)}} - q'(p)\, q(p).$$

It follows that for any $p > c$, $M(p) > -q'(p)\, \pi'(p) \geq 0$, i.e. $M(p) > 0$. Then for any $p > c$, from (A.24), $\phi(p) - v(c) + v(p)$ is strictly increasing, which implies that $\phi(p) > v(c) - v(p)$.

Proof of Corollary 2. We show that $s_{o,s} \leq s_{o,l}\left(p_o^2, c, F_{n,l}^2\right)$. Through the entire proof, we always refer to the optimal prices. Remember that p_o^2 is such that

$$2t - v(c) = \psi\left(p_o^2\right),$$

where $\psi(p) \equiv 2\phi(p) - v(p)$, $F_{n,s}^2 = \frac{t}{3}$, $s_{o,s} = \frac{1}{3}$,

$$F_{n,l}^2 = t - \phi(p_o^2), \quad \text{and} \quad s_{o,l}\left(p_o^2, c, F_{n,l}^2\right) = \frac{\phi(p_o^2)}{2t}.$$

Therefore, $s_{o,s} \leq s_{o,l}\left(p_0^2, c, F_{n,l}^2\right)$ is equivalent to $F_{n,l}^2 \leq F_{n,s}^2$, which again is equivalent to

$$\frac{2}{3}t \leq \phi(p_o^2).$$

Note that as $t \to 0^+$, we know that $p_0^2 \to c$; and since $\phi(c) = 0$, the result is true at $t = 0$. If we differentiate both sides with respect to t,

$$\frac{2}{3} \leq \phi'(p_o^2) \cdot \frac{2}{2\phi'(p_o^2) + q(p_o^2)} \iff q(p_o^2) \leq \phi'(p_o^2), \quad (*)$$

where $\frac{\partial p_o^2}{\partial t} = \frac{2}{2\phi'(p_o^2)+q(p_o^2)}$. By Lemma A1-(i), inequality $(*)$ is true whenever $p_o^2 > c$, which is true as long as $0 < t < v(c)$.

Proof of Proposition 4. Throughout the proof, we consider only the symmetric equilibrium and therefore omit some unnecessary subscripts. Let us recall that in equilibrium, the long-term membership, p_o^2, is

$$2t - v(c) = \psi\left(p_o^2\right), \tag{A.25}$$

where $\psi(p) \equiv 2\phi(p) - v(p)$ and $\psi'(\cdot) > 0$, so p_o^2 is uniquely defined by $\psi^{-1}(2t - v(c))$. Since $t \in (0, v(c))$, it follows that $p_o^2 > c$ and $\psi(p_o^2) > \psi(c)$. The optimal membership fees for new customers in period 1 and 2 are

$$F_l = t + \delta \frac{t \cdot q(p_o^2)}{2\phi'(p_o^2) + q(p_o^2)} + \delta \frac{\phi'(p_o^2) + q(p_o^2)}{2\phi'(p_o^2) + q(p_o^2)} \left[v(c) - v(p_o^2) - \pi(p_o^2)\right] \tag{A.26}$$

and

$$F_{n,l}^2 = \frac{v(c) - v(p_o^2)}{2}, \tag{A.27}$$

respectively. The market share in period 2 for old customers for firm A is

$$s_{o,l}\left(p_o^2, c, F_l^2\right) = \frac{1}{2} + \frac{v(p_o^2) - v(c)}{4t} < \frac{1}{2}, \tag{A.28}$$

where $p_o^2 \equiv p_{o,l}^2$, to simplify notation. Using (A.25) in the profit function, in equilibrium

$$\begin{aligned}\pi_l(t) = \frac{t}{2} + \frac{\delta t}{2} \cdot \frac{q(p_o^2)}{2\phi'(p_o^2) + q(p_o^2)} + \frac{\delta}{2} \cdot \frac{\phi'(p_o^2) + q(p_o^2)}{2\phi'(p_o^2) + q(p_o^2)} \left[v(c) - v(p_o^2) - \pi(p_o^2)\right] \\ +\delta\left[\pi(p_o^2)\left(\frac{1}{2} + \frac{v(p_o^2) - v(c)}{4t}\right) + \frac{(v(c) - v(p_o^2))^2}{8t}\right].\end{aligned} \tag{A.29}$$

Similarly, remember that the optimal membership fees for old and new customers in period 2 are $F_s = \frac{3+\delta}{3}t$ and $F_{n,s}^2 = \frac{t}{3}$, and market share for the old customers is given by $s_{o,s} = \frac{1}{3}$. The short-term membership equilibrium profit is then equal to

$$\pi_s(t) = \frac{t}{18}(9+8\delta). \tag{A.30}$$

Let $V \equiv v(c) - v(p_o^2)$. Given that $t = \frac{1}{2}(2\phi(p_o^2) + V)$, it follows that

$$\frac{\partial p_o^2}{\partial t} \equiv \left(p_o^2\right)_t = \frac{2}{2\phi'(p_o^2) + q(p_o^2)}.$$

Then,

$$\pi_s(t) - \pi_l(t) =$$
$$\frac{\delta}{8t}\left\{\frac{1}{4}\left(2\phi\left(p_o^2\right) + V\right)\left(\frac{136}{9}\phi\left(p_o^2\right) - \frac{40}{9}V + 4\pi\left(p_o^2\right) - 2q(p_o^2)\left(p_o^2\right)_t L\right)\right.$$
$$\left. -4\phi\left(p_o^2\right)\left[\pi\left(p_o^2\right) + \phi\left(p_o^2\right)\right]\right\},$$

where $L = (2\phi(p_o^2) + 2V - \pi(p_o^2))$. Using Lemma A1-(i), for $p > c$

$$\phi'(p) > q(p) \implies \frac{2q(p_o^2)}{2\phi'(p_o^2) + q(p_o^2)} \leq \frac{2}{3} \implies q(p_o^2)\left(p_o^2\right)_t \leq \frac{2}{3}.$$

Therefore

$$\pi_s(t) - \pi_l(t) \geq \frac{\delta}{18t}\left[\phi\left(p_o^2\right) - V\right]\left(3\left(\phi\left(p_o^2\right) - \pi\left(p_o^2\right)\right) + 4V + 2\phi\left(p_o^2\right)\right) \geq 0. \tag{A.31}$$

Note that (A.31) is positive, since $\phi(p_o^2) \geq \pi(p_o^2)$, $V = v(c) - v(p_o^2) \geq 0$, $\phi(p_o^2) \geq 0$, and $\phi(p_o^2) - V \geq 0$ by Lemma A1-(ii).

Lemma A2. *Given (A2) and $V \equiv v(c) - v(p)$, for any $p > c$*

- (i) $(p-c)\left[q(p)\phi(p)\right]' - \phi(p)\phi'(p) < 0$;
- (ii) *if additionally, $p < c^*$ ($c^* > c$ is defined in the proof), then* $(5\phi'(p) - 2q(p))(V + 2\phi(p)) - 9\phi(p)\phi'(p) > 0$;
- (iii) $\phi(p)\left[3\pi(p) - V - \phi(p)\right] - V^2 < 0$;
- (iv) $\phi(p)(\pi(p) - V) + 2V(\phi(p) - V) > 0$.

Proof of Lemma A2. (i) Recall $\phi(p) \equiv \frac{q(p)\pi(p)}{\pi'(p)}$. Let $p > c$, then

$$\begin{aligned}
&(p-c)\left[q(p)\phi(p)\right]' - \phi(p)\phi'(p) \\
&= \frac{(p-c)^2 q(p)^2 q'(p)\left(3(p-c)^2 q'(p)^2 - (p-c)q(p)\left((p-c)q''(p) - 3q'(p)\right) + 2q(p)^2\right)}{\pi'(p)^3} \\
&= \frac{(p-c)^2 q(p)^2 q'(p) M(p)}{\pi'(p)^3},
\end{aligned}$$

where $M(p) \equiv 3(p-c)^2 q'(p)^2 - (p-c)q(p)\left((p-c)q''(p) - 3q'(p)\right) + 2q(p)^2$. Note that,

$$(p-c)\left[q\left(p\right)\phi\left(p\right)\right]'-\phi\left(p\right)\phi'\left(p\right)<0\iff M(p)>0. \tag{A.32}$$

If we show that for any $p>c$, $M(p)>0$, then the proof of (i) will be concluded. The term $M(p)$ can be rewritten as

$$M(p)=q'\left(p\right)^2\left(p-c\right)^2\underbrace{\left\{3-\frac{q\left(p\right)q''\left(p\right)}{q'\left(p\right)^2}\right\}}_{>1,\text{ by (A2)}}+3\left(p-c\right)q'\left(p\right)q\left(p\right)+2q\left(p\right)^2. \tag{A.33}$$

Given (A2), from (A.33)

$$M(p)>\pi'\left(p\right)\left[\pi'\left(p\right)+q\left(p\right)\right].$$

Note that for any $p>c$, $\pi'\left(p\right)\left[\pi'\left(p\right)+q\left(p\right)\right]>0$, which implies that $M(p)>0$.

(ii) Note that for any $p>c$

$$\left(5\phi'\left(p\right)-2q\left(p\right)\right)\left(V+2\phi\left(p\right)\right)-9\phi\left(p\right)\phi'\left(p\right)=$$

$$\underbrace{\left(\phi'\left(p\right)-q\left(p\right)\right)}_{>0,\text{ by Lemma A1-(i)}}\left(\phi\left(p\right)+2V\right)+3\left(\phi'\left(p\right)V-q\left(p\right)\phi\left(p\right)\right).$$

If we show that $\phi'\left(p\right)V-q\left(p\right)\phi\left(p\right)\geq 0$ the proof will be completed. First notice that at $p=c$, $\phi'\left(c\right)V-q\left(c\right)\phi\left(c\right)=0$, and

$$\left[\phi'\left(p\right)V-q\left(p\right)\phi\left(p\right)\right]'=\phi''\left(p\right)V-q'\left(p\right)\phi\left(p\right). \tag{A.34}$$

We want to show that the left-hand side of (A.34) is strictly positive for any $p>c$. It is enough to show that $\phi''(p)\geq 0$. By (A2) and the fact that $\phi(p)\equiv\frac{q(p)\pi(p)}{\pi'(p)}$,

$$\lim_{p\to c^+}\frac{\pi'\left(p\right)^3\phi''\left(p\right)}{\left(p-c\right)}=3q\left(c\right)^2q'\left(c\right)^2\left(2-\frac{q''\left(c\right)q\left(c\right)}{q'\left(c\right)^2}\right)>0.$$

By continuity there exists $c^*>c$ such that if $p\in(c,c^*)$, then $\phi''(p)>0$. From (A.34), for any $p\in(c,c^*)$, $\phi'\left(p\right)V-q\left(p\right)\phi\left(p\right)$ is strictly increasing in p. Thus, for any $p\in(c,c^*)$, $\phi'\left(p\right)V-q\left(p\right)\phi\left(p\right)>0$.

(iii) Notice that $\phi\left(p\right)\left[3\pi\left(p\right)-V-\phi\left(p\right)\right]-V^2$ can be rewritten as

$$\phi\left(p\right)\left[\underbrace{\left(\pi\left(p\right)-V\right)}_{<0}-\underbrace{\left(\phi\left(p\right)-\pi\left(p\right)\right)}_{>0}\right]+\left(\phi\left(p\right)\pi\left(p\right)-V^2\right).$$

If we can show that $\pi(p)\phi(p) - V^2 \leq 0$ for any $p > c$, then (iii) will be proved. The inequality $\pi(p)\phi(p) - V^2 \leq 0$ is equivalent to

$$\pi(p)^2 q(p) - \pi'(p) V^2 \leq 0.$$

Note that $\pi(c)^2 q(c) - \pi'(c) V^2 = 0$ at $p = c$, and its derivative is given by

$$\text{(A.35)} \qquad \left[\pi(p)^2 q(p) - \pi'(p) V^2\right]' = 2\pi'(p) q(p) \underbrace{[\pi(p) - V]}_{\leq 0} + \pi(p)^2 q'(p) - \pi''(p) V^2 < 0.$$

From (A.35), $\pi(p)^2 q(p) - \pi'(p) V^2$ is strictly decreasing for any $p > c$. Thus, $\pi(p)^2 q(p) - \pi'(p) V^2 < 0$ for any $p > c$. Concluding the proof of (iii).

(iv) Let $p > c$. Note that

$$\phi(p)(\pi(p) - V) + 2V(\phi(p) - V) = \phi(p)\pi(p) + V\phi(p) - 2V^2.$$

If we can show that $V(\phi(p) - V) > V^2 - \phi(p)\pi(p)$, then the proof of (iv) will be completed. Note that,

$$\text{(A.36)} \qquad \begin{aligned} V(\phi(p) - V) > V^2 - \phi(p)\pi(p) \iff \\ \phi(p) - V > V - \frac{\phi(p)\pi(p)}{V}. \end{aligned}$$

By Lemma A1-(ii), $V - \frac{\phi(p)\pi(p)}{V} < V - \pi(p)$. Let us show that $\phi(p) - V > V - \pi(p)$. At $p = c$, $\phi(c) + \pi(c) - 2V = 0$, and

$$\text{(A.37)} \qquad [\phi(p) + \pi(p) - 2V]' = \frac{(p-c)^2 \overbrace{\left((p-c) q'(p)^3 - q(p)^2 q''(p) + 3q(p) q'(p)^2\right)}^{\equiv M(p)}}{\pi'(p)^2}.$$

Such a derivative is positive because, by (A2), for $p > c$

$$M(p) = q'(p)^2 \pi'(p) + q(p) q'(p)^2 \left(2 - \frac{q(p) q''(p)}{q'(p)^2}\right) > 0.$$

It follows that $\phi(p) + \pi(p) - 2V > 0$ for any $p > c$, and the proof of (iv) is concluded.

Proof of Proposition 5. Several steps on this proof are similar to those in the proofs of Propositions 1 and 3, so we omit them.

(ii-a) and (ii-b): Firms A and B maximize second-period profits on A's turf by setting $p^{2*}_{A,o} = p^{2*}_{B,n} = c$,

$$\text{(A.38)} \qquad F^{2*}_{A,o} = \frac{1}{3} t(2x^* + 1), \quad \text{and} \quad F^{2*}_{B,n} = \frac{1}{3} t(4x^* - 1).$$

Similarly, both firms maximize second-period profits on B's turf by setting $p^{2*}_{A,n} = c$,

$$F^{2*}_{A,n} = \frac{1}{2}\left(t\left(1-2x^*\right)+v\left(c\right)-v\left(p^{2*}_{B,o}\right)\right), \text{ and}$$
$$p^{2*}_{B,o} = \psi^{-1}\left(t\left(3-2x^*\right)-v\left(c\right)\right), \tag{A.39}$$

where $\psi(p) \equiv 2\phi(p) - v(p)$ and $\phi(p) \equiv \frac{q(p)\pi(p)}{\pi'(p)}$. Note that $p^{2*}_{B,o} > c$ given that $\psi'(\cdot) > 0$. Which concludes the proofs of (ii-a) and (ii-b).

(i) and (iii): We characterize the equilibrium values of p_i and F_i for $i \in \{A, B\}$. Then, we show the existence of a unique cutoff $x^* > 1/2$ satisfying (5.2) in equilibrium.

The problem of firm A in period 1 is

$$\max_{p_A,F_A} \quad x^*\left(\alpha\right)\left(\pi\left(p_A\right)+F_A\right)+\delta s_A\left(c,F^{2*}_{A,o},c,F^{2*}_{B,n}\right)F^{2*}_{A,o}$$
$$+\,\delta\left[s_B\left(p^{2*}_{B,o},c,F^{2*}_{A,n}\right)-x^*\left(\alpha\right)\right]F^{2*}_{A,n}, \tag{A.40}$$

and for firm B is

$$\max_{p_B,F_B} \quad \left(1-x^*\left(\alpha\right)\right)\left(\pi\left(p_B\right)+F_B\right)+\delta\left(1-s_B\left(p^{2*}_{B,o},c,F^{2*}_{A,n}\right)\right)\pi\left(p^{2*}_{B,o}\right)$$
$$+\,\delta\left[x^*\left(\alpha\right)-s_A\left(c,F^{2*}_{A,o},c,F^{2*}_{B,n}\right)\right]F^{2*}_{B,n}. \tag{A.41}$$

Both, (A.40) and (A.41) are strictly concave in F_A and F_B, respectively. From (5.2),

$$\frac{\partial x^*\left(\alpha\right)}{\partial p_A} = q\left(p_A\right)\frac{\partial x^*\left(\alpha\right)}{\partial F_A} \quad \text{and} \quad \frac{\partial x^*\left(\alpha\right)}{\partial p_B} = q\left(p_B\right)\frac{\partial x^*\left(\alpha\right)}{\partial F_B}. \tag{A.42}$$

From the first-order conditions of (A.40) and (A.41) and (A.42), it follows that

$$q'\left(p_A\right)\left(p_A-c\right)x^*\left(\alpha\right)=0 \quad \text{and} \quad q'\left(p_B\right)\left(p_B-c\right)\left(1-x^*\left(\alpha\right)\right)=0. \tag{A.43}$$

Note that from (5.2), $\frac{\partial x^*(\alpha)}{\partial p_A} < 0$ and $\frac{\partial x^*(\alpha)}{\partial p_B} > 0$. It follows from (A.43), that (A.40) and (A.41) are single-peaked in p_A and p_B, respectively, and reach a unique maximum at $p_A = c$ and $p_B = c$, respectively. In equilibrium, F_A and F_B are equal to

$$F^*_A = \frac{t}{18}\left(12\left(3-2\delta\right)x^*-4\delta\left(1-x^*\right)\right)+\delta\left(t+v\left(c\right)-v\left(p^{2*}_{B,o}\right)\right)\frac{\phi'\left(p^{2*}_{B,o}\right)+q\left(p^{2*}_{B,o}\right)}{2\phi'\left(p^{2*}_{B,o}\right)+q\left(p^{2*}_{B,o}\right)}, \tag{A.44}$$

and

$$F^*_B = \frac{t}{9}\left(\delta\left(13x^*-1\right)+18\left(1-x^*\right)\right)+\delta\frac{t\left(1-x^*\right)q\left(p^{2*}_{B,o}\right)-\left[\pi\left(p^{2*}_{B,o}\right)\phi\left(p^{2*}_{B,o}\right)\right]'}{2\phi'\left(p^{2*}_{B,o}\right)+q\left(p^{2*}_{B,o}\right)}, \tag{A.45}$$

respectively. Let us show the existence of a unique cutoff $x^* > 1/2$ such that (5.2) is satisfied in equilibrium From (A.39),

$$t = \frac{1}{3-2x^*}\left(2\phi\left(p_{B,o}^{2*}\right) + v\left(c\right) - v\left(p_{B,o}^{2}\right)\right). \tag{A.46}$$

Plugging (A.46) into (A.44), (A.45), and (5.2) yields

$$x^* = \frac{N}{D}, \tag{A.47}$$

where

$$\begin{aligned} N \equiv\ & 27\delta\left(p_{B,o}^{2*} - c\right)\left[\phi\left(p_{B,o}^{2*}\right) q\left(p_{B,o}^{2*}\right)\right]' \\ & - \left(\left(27-14\delta\right) q\left(p_{B,o}^{2*}\right) + 2\left(27-5\delta\right)\phi'\left(p_{B,o}^{2*}\right)\right)\left[v(c) - v\left(p_{B,o}^{2*}\right) + 2\phi\left(p_{B,o}^{2*}\right)\right], \end{aligned} \tag{A.48}$$

and

$$\begin{aligned} D \equiv\ & 18\delta\left(p_{B,o}^{2*} - c\right)\left[\phi\left(p_{B,o}^{2*}\right) q\left(p_{B,o}^{2*}\right)\right]' \\ & - 2\left(27-10\delta\right)\left(q\left(p_{B,o}^{2*}\right) + 2\phi'\left(p_{B,o}^{2*}\right)\right)\left[v(c) - v\left(p_{B,o}^{2*}\right) + 2\phi\left(p_{B,o}^{2*}\right)\right]. \end{aligned} \tag{A.49}$$

From (A.39), $p_{B,o}^{2*}$ is a function of x^*, which implies that both $N = N(x^*)$ and $D = D(x^*)$ are functions of x^*. Note that (A.47) implicitly defines x^*. We need to find a fixed point of the function $g : [0,1] \longrightarrow \mathbb{R}$, defined as $g(x^*) \equiv N(x^*)/D(x^*)$. Note that for any $x^* \in [0,1]$, by Lemma A2-(i), both $N(x^*)$ and $D(x^*)$ are strictly negative, so that $g(x^*) > 0$. Also, note that

$$\begin{aligned} N(x^*) - D(x^*) =\ & 9\delta\left(p_{B,o}^{2*} - c\right)\left[\phi\left(p_{B,o}^{2*}\right) q\left(p_{B,o}^{2*}\right)\right]' \\ & + 3\left[\left(9-2\delta\right) q\left(p_{B,o}^{2*}\right) + 2\left(9-5\delta\right)\phi'\left(p_{B,o}^{2*}\right)\right]\underbrace{\left(v(c) - v\left(p_{B,o}^{2*}\right) + 2\phi\left(p_{B,o}^{2*}\right)\right)}_{>0,\text{ since } p_{B,o}^{2*} > c.} \geq 0, \end{aligned}$$

which implies that $g(x^*) \leq 1$ for any $x^* \in [0,1]$. It follows that $g : [0,1] \longrightarrow [0,1]$, and by *Brouwer's fixed-point theorem* there is a solution, $x^* \in [0,1]$, of (A.47). Note that $x^* > 1/2$ if and only if $2N(x^*) < D(x^*)$, which is equivalent to

$$\begin{aligned} & 9\underbrace{\left(\phi\left(p_{B,o}^{2*}\right)\phi'\left(p_{B,o}^{2*}\right) - \left(p_{B,o}^{2*} - c\right)\left[\phi\left(p_{B,o}^{2*}\right) q\left(p_{B,o}^{2*}\right)\right]'\right)}_{>0,\text{ by Lemma A2-(i)}} \\ & + \underbrace{\left(5\phi'\left(p_{B,o}^{2*}\right) - 2q\left(p_{B,o}^{2*}\right)\right)\left(v(c) - v\left(p_{B,o}^{2*}\right) + 2\phi\left(p_{B,o}^{2*}\right)\right) - 9\phi\left(p_{B,o}^{2*}\right)\phi'\left(p_{B,o}^{2*}\right)}_{>0,\text{ by Lemma A2-(ii)}} > 0. \end{aligned} \tag{A.50}$$

From Lemma A2(i and ii), (A.50) holds true as long as $p_{B,o}^{2*} \in (c, c^*)$. From (A.39), $p_{B,o}^{2*} \to c$ as $t \to 0$, by continuity there exists $\underline{t} \in (0, v(c))$ such that for any $t \in (0, \underline{t})$: $p_{B,o}^{2*} \in (c, c^*)$. Thus, $x^* > 1/2$. Uniqueness follows by showing that when $t \to 0$, $|g'(x^*)| < 1$, which implies that g is a contracting mapping, and by *Banach's fixed point theorem* the solution of (A.47) must be unique. Let us show that $|g'(x^*)| < 1$ when $t \to 0$. From (A.47), for any $x^* \in [^1\!/_2, 1]$,

$$g'(x^*) = \frac{N'(x^*)D(x^*) - D'(x^*)N(x^*)}{D(x^*)^2}. \tag{A.51}$$

It is straightforward to show that as $t \to 0$, $p^{2*}_{B,o} \to c$, $N'(x^*)D(x^*) - D'(x^*)N(x^*) \to 1620\delta(11\delta - 27)q(c)^3 q'(c)$, and $D(x^*)^2 \to 648(27 - 11\delta)^2 q(c)^4$. Thus,

$$g'(x^*) \to -\frac{5\delta q'(c)}{2(27 - 11\delta)q(c)} > 0.$$

The latter expression is strictly less than 1 since ${}^{-q'(c)}\!/_{q(c)} < 6.4 = \min_{\delta\in[0,1]} {}^{2(27-11\delta)}\!/_{5\delta}$. Finally, by the continuity of the derivatives of (A.48) and (A.49), there exists $\underline{t}_2$ such that $g'(x^*) < 1$ for any $t < \underline{t}_2$. Thus, $t^* \equiv \min\{\underline{t}, \underline{t}_2\}$.

Proof of Corollary 3. The proof of items (i)-(iv) can be divided into:

(a) $F^{2*}_{B,n} > F^{2*}_{A,n}$ and $F^*_B > F^*_A$;
(b) $F^{2*}_{A,o} > \pi\left(p^{2*}_{B,o}\right)$;
(c) $s_A\left(c, F^{2*}_{A,o}, c, F^{2*}_{B,n}\right) > 1 - s_B\left(p^{2*}_{B,o}, c, F^{2*}_{A,n}\right)$ if and only if $\delta > {}^{27}\!/_{41}$;
(d) $x^* - s_A\left(c, F^{2*}_{A,o}, c, F^{2*}_{B,n}\right) > s_B\left(p^{2*}_{B,o}, c, F^{2*}_{A,n}\right) - x^*$;
(e) $\pi_{A,s}(t) > \pi_{B,l}(t)$.

Note that (i) and (iv) follow from (a) and (e); (ii) follows from (b) and (c), and (iii) follows from (a) and (d).

(a) Let us begin by proving that $F^*_B > F^*_A$. Plugging (A.47) into (A.44) and (A.45)

$$\begin{aligned} \frac{1}{\delta} D_2 \cdot \left(F^*_B - F^*_A\right) = {} & 3\left(\delta+3\right) \underbrace{\left[\left(p^{2*}_{B,o} - c\right)\left(q\left(p^{2*}_{B,o}\right)\phi\left(p^{2*}_{B,o}\right)\right)' - \phi\left(p^{2*}_{B,o}\right)\phi'\left(p^{2*}_{B,o}\right)\right]}_{<0,\text{ by Lemma A2-(i)}} \\ & + 2\left(\delta+3\right)\phi\left(p^{2*}_{B,o}\right) \underbrace{\left[q\left(p^{2*}_{B,o}\right) - \phi'\left(p^{2*}_{B,o}\right)\right]}_{<0,\text{ by Lemma A1-(i)}} \\ & + \underbrace{\left(V - \phi\left(p^{2*}_{B,o}\right)\right)}_{<0,\text{ by Lemma A1-(ii)}} \left[\left(11-2\delta\right)q\left(p^{2*}_{B,o}\right) + \left(13-10\delta\right)\phi'\left(p^{2*}_{B,o}\right)\right], \end{aligned} \tag{A.52}$$

where

$$D_2 \equiv \left(25\delta - 54\right)\phi'\left(p^{2*}_{B,o}\right) + \left(8\delta - 27\right)q\left(p^{2*}_{B,o}\right) < 0.$$

From Lemma A1 and Lemma A2 the right-hand side of (A.52) is negative. It follows that $F^*_B > F^*_A$. To prove that $F^{2*}_{B,n} > F^{2*}_{A,n}$, from (A.38), (A.39) and (A.47) it follows that

$$\underbrace{\left((25\delta-54)\,\phi'\left(p^{2}_{B,o}\right)+(8\delta-27)\,q\left(p^{2}_{B,o}\right)\right)}_{<0}\cdot\left(F^{2*}_{B,n}-F^{2*}_{A,n}\right)=$$

$$12\delta\underbrace{\left\{\left(p^{2*}_{B,o}-c\right)\left[q\left(p^{2*}_{B,o}\right)\phi\left(p^{2*}_{B,o}\right)\right]'-\phi\left(p^{2*}_{B,o}\right)\phi'\left(p^{2*}_{B,o}\right)\right\}}_{<0,\text{ by Lemma A2-(i)}}$$

$$-\underbrace{\left(9q\left(p^{2*}_{B,o}\right)+(18-15\delta)\,\phi'\left(p^{2*}_{B,o}\right)\right)\left(\phi\left(p^{2*}_{B,o}\right)-V\right)}_{>0,\text{ by Lemma A1-(ii)}}+8\delta\phi\left(p^{2*}_{B,o}\right)\underbrace{\left(q\left(p^{2*}_{B,o}\right)-\phi'\left(p^{2*}_{B,o}\right)\right)}_{<0,\text{ by Lemma A1-(i)}}.$$

Thus, from Lemma A1 and Lemma A2, $F^{2*}_{B,n}>F^{2*}_{A,n}$, concluding the proof of (a).

(b) Let us prove that $F^{2*}_{A,o}-\pi\left(p^{2*}_{B,o}\right)>0$. From (A.38), (A.39) and (A.47),

(A.53)
$$\left((25\delta-54)\phi'\left(p^{2*}_{B,o}\right)+(8\delta-27)q\left(p^{2*}_{B,o}\right)\right)\cdot\left(F^{2*}_{A,o}-\pi\left(p^{2*}_{B,o}\right)\right)=$$
$$6\delta\left(p^{2*}_{B,o}-c\right)\left[\phi\left(p^{2*}_{B,o}\right)q\left(p^{2*}_{B,o}\right)\right]'+q\left(p^{2*}_{B,o}\right)\left(p^{2*}_{B,o}-c\right)\left((27-8\delta)\,q\left(p^{2*}_{B,o}\right)+(54-25\delta)\,\phi'\left(p^{2*}_{B,o}\right)\right)$$
$$-\left((9-4\delta)\,q\left(p^{2*}_{B,o}\right)+(18-5\delta)\,\phi'\left(p^{2*}_{B,o}\right)\right)\left(v(c)-v\left(p^{2*}_{B,o}\right)+2\phi\left(p^{2*}_{B,o}\right)\right).$$

It follows that $F^{2*}_{A,o}-\pi\left(p^{2*}_{B,o}\right)$ can be writen as a fraction $NF(p^{2*}_{B,o})/DF(p^{2*}_{B,o})$. From L'Hôpital's rule, as $t\to 0$, $p^{2*}_{B,o}\to c$ and

$$\left(F^{2*}_{A,o}-\pi\left(p^{2*}_{B,o}\right)\right)\Big|_{p^{2*}_{B,o}=c}=0,$$

$$\frac{\partial}{\partial p^{2*}_{B,o}}\left(F^{2*}_{A,o}-\pi\left(p^{2*}_{B,o}\right)\right)\Big|_{p^{2*}_{B,o}=c}=0,$$

$$\frac{\partial^2}{\partial\left(p^{2*}_{B,o}\right)^2}\left(F^{2*}_{A,o}-\pi\left(p^{2*}_{B,o}\right)\right)\Big|_{p^{2*}_{B,o}=c}=-\frac{5\,(27-5\delta)\,q'\,(c)}{3\,(27-11\delta)}>0.$$

By continuity there exists $t_b\in(0,v(c))$ such that for any $t\in(0,t_b)$, thus, (b) holds true.

(c) $s_A\left(c,F^{2*}_{A,o},c,F^{2*}_{B,n}\right)>1-s_B\left(p^{2*}_{B,o},c,F^{2*}_{A,n}\right)$ if and only if $\delta>27/41$. From (A.38), (A.39) and (A.47), there are functions $N^s_{AB}(p^{2*}_{B,o})$ and $D^s_{AB}(p^{2*}_{B,o})$ such that

$$s_A\left(c,F^{2*}_{A,o},c,F^{2*}_{B,n}\right)-1+s_B\left(p^{2*}_{B,o},c,F^{2*}_{A,n}\right)=\frac{N^s_{AB}(p^{2*}_{B,o})}{D^s_{AB}(p^{2*}_{B,o})},\tag{A.54}$$

where

$$N^s_{AB}(p^{2*}_{B,o}) \equiv 6\delta \left\{ \left(p^{2*}_{B,o} - c\right) \left[\phi\left(p^{2*}_{B,o}\right) q\left(p^{2*}_{B,o}\right)\right]' - \phi'\left(p^{2*}_{B,o}\right) \phi\left(p^{2*}_{B,o}\right) \right\}$$
$$+ 9\left(q\left(p^{2*}_{B,o}\right) + (2-\delta)\,\phi'\left(p^{2*}_{B,o}\right)\right) \phi\left(p^{2*}_{B,o}\right) \tag{A.55}$$
$$- \left((9-4\delta)\, q\left(p^{2*}_{B,o}\right) + (18-5\delta)\,\phi'\left(p^{2*}_{B,o}\right)\right)\left(v(c) - v\left(p^{2*}_{B,o}\right)\right),$$

and

(A.56)

$$D^s_{AB}(p^{2*}_{B,o}) \equiv$$
$$9\delta\left(p^{2*}_{B,o} - c\right)\left[\phi\left(p^{2*}_{B,o}\right) q\left(p^{2*}_{B,o}\right)\right]' + (10\delta - 27)\left(q\left(p^{2*}_{B,o}\right) + 2\phi'\left(p^{2*}_{B,o}\right)\right)\left(v(c) - v\left(p^{2*}_{B,o}\right) + 2\phi\left(p^{2*}_{B,o}\right)\right).$$

From L'Hôpital's rule, as $t \to 0$, $p^{2*}_{B,o} \to c$ and

$$\left(s_A\left(c, F^{2*}_{A,o}, c, F^{2*}_{B,n}\right) - 1 + s_B\left(p^{2*}_{B,o}, c, F^{2*}_{A,n}\right)\right)\Big|_{p^{2*}_{B,o}=c} = 0,$$

$$\frac{\partial}{\partial p^{2*}_{B,o}}\left[s_A\left(c, F^{2*}_{A,o}, c, F^{2*}_{B,n}\right) - 1 + s_B\left(p^{2*}_{B,o}, c, F^{2*}_{A,n}\right)\right]\Big|_{p^{2*}_{B,o}=c} = -\frac{(41\delta - 27)\, q'(c)}{18\,(27 - 11\delta)\, q(c)} > 0 \iff$$

$\delta > 27/41$. By continuity there exists $t_c \in (0, v(c))$ such that for any $t \in (0, t_c)$, (c) holds true.

(d) Let us show that $x^* - s_A\left(c, F^{2*}_{A,o}, c, F^{2*}_{B,n}\right) > s_B\left(p^{2*}_{B,o}, c, F^{2*}_{A,n}\right) - x^*$. From (A.38), (A.39) and (A.47), there are functions $N^s(p^{2*}_{B,o})$ and $D^s(p^{2*}_{B,o})$ such that

$$2x^* - s_A\left(c, F^{2*}_{A,o}, c, F^{2*}_{B,n}\right) - s_B\left(p^{2*}_{B,o}, c, F^{2*}_{A,n}\right) = \frac{N^s(p^{2*}_{B,o})}{D^s(p^{2*}_{B,o})}, \tag{A.57}$$

where

(A.58)

$$N^s(p^{2*}_{B,o}) \equiv$$
$$12\delta\left(p^{2*}_{B,o} - c\right)\left[\phi\left(p^{2*}_{B,o}\right) q\left(p^{2*}_{B,o}\right)\right]' + \left(9q\left(p^{2*}_{B,o}\right) + 3\,(6-5\delta)\,\phi'\left(p^{2*}_{B,o}\right)\right)\left(v(c) - v\left(p^{2*}_{B,o}\right)\right)$$
$$+ \left((8\delta - 9)\, q\left(p^{2*}_{B,o}\right) - (5\delta + 18)\,\phi'\left(p^{2*}_{B,o}\right)\right)\phi\left(p^{2*}_{B,o}\right),$$

and

(A.59)

$$D^s(p^{2*}_{B,o}) \equiv$$
$$9\delta\left(p^{2*}_{B,o} - c\right)\left[\phi\left(p^{2*}_{B,o}\right) q\left(p^{2*}_{B,o}\right)\right]' + (10\delta - 27)\left(q\left(p^{2*}_{B,o}\right) + 2\phi'\left(p^{2*}_{B,o}\right)\right)\left(v(c) - v\left(p^{2*}_{B,o}\right) + 2\phi\left(p^{2*}_{B,o}\right)\right)$$

From L'Hôpital's rule, as $t \to 0$, $p^{2*}_{B,o} \to c$ and

$$\left(2x^{*}-s_{A}\left(c,F_{A,o}^{2*},c,F_{B,n}^{2*}\right)-s_{B}\left(p_{B,o}^{2*},c,F_{A,n}^{2*}\right)\right)\Big|_{p_{B,o}^{2*}=c}=0,$$

$$\frac{\partial}{\partial p_{B,o}^{2*}}\left[2x^{*}-s_{A}\left(c,F_{A,o}^{2*},c,F_{B,n}^{2*}\right)-s_{B}\left(p_{B,o}^{2*},c,F_{A,n}^{2*}\right)\right]\Big|_{p_{B,o}^{2*}=c}=-\frac{\left(49\delta+27\right)q'\left(c\right)}{18\left(27-11\delta\right)q\left(c\right)}>0,$$

By continuity there exists $t_d\in(0,v(c))$ such that for any $t\in(0,t_d)$, (b) holds true.

(e) Let $\pi^*_{A,s}(t)$ and $\pi^*_{B,l}(t)$ be equal to the equilibrium profits for firm A and B, respectively. We want to show that $\pi_{A,s}(t)-\pi_{B,l}(t)>0$. First note that

$$\begin{aligned}\pi^*_{A,s}(t)-\pi^*_{B,l}(t)&=\left(x^*F^*_A-\left(1-x^*\right)F^*_B\right)+\frac{2}{3}\delta t\left(1-x^*\right)x^*\\&\quad+\frac{1}{8t}\delta\left(t\left(1-2x^*\right)+v\left(c\right)-v\left(p_{B,o}^{2*}\right)\right)^2-\frac{1}{2t}\delta\phi\left(p_{B,o}^{2*}\right)\pi\left(p_{B,o}^{2*}\right).\end{aligned}$$

From (A.39), $p_{B,o}^{2*}\to c$ as $t\to 0$. Also, from (A.44), (A.45) and (A.47)

$$x^*F^*_A-\left(1-x^*\right)F^*_B=\frac{N_{AB}(p_{B,n}^{2*})}{D_{AB}(p_{B,n}^{2*})},\tag{A.60}$$

and as $t\to 0$, $p_{B,o}^{2*}\to c$, $N_{AB}(p_{B,n}^{2*})\to 0$, $D_{AB}(p_{B,n}^{2*})\to 0$, thus, from (A.60)

$$\left(x^*F^*_A-\left(1-x^*\right)F^*_B\right)\Big|_{p_{B,o}^{2*}=c}=0.$$

Note, the derivative of the left-hand side of (A.60) with respect to $p_{B,o}^{2}$ is

$$\frac{\partial}{\partial p_{B,o}^{2*}}\left(x^*F^*_A-\left(1-x^*\right)F^*_B\right)=\frac{N'_{AB}(p_{B,n}^{2*})D_{AB}(p_{B,n}^{2*})-D'_{AB}(p_{B,n}^{2*})N_{AB}(p_{B,n}^{2*})}{D_{AB}(p_{B,n}^{2*})^2}.\tag{A.61}$$

From (A.61) and the L'Hôpital's rule it follows that

$$\frac{\partial}{\partial p_{B,o}^{2*}}\left(x^*F^*_A-\left(1-x^*\right)F^*_B\right)\Big|_{p_{B,o}^{2*}=c}=\frac{\delta^2\left(27-11\delta\right)q\left(c\right)}{2\left(27-10\delta\right)^2}>0.$$

By continuity there exists $t_e\in(0,v(c))$ such that for any $t\in(0,t_e)$, $x^*F^*_A-\left(1-x^*\right)F^*_B>0$. Now, let $P_2\equiv\pi^*_{A,s}-\pi^*_{B,l}-\left(x^*F^*_A-\left(1-x^*\right)F^*_B\right)$, and let us show that $P_2\geq 0$ for any $p_{B,o}^{2*}>c$. Plugging (A.47) into P_2

$$\frac{1}{\delta}D_3 \cdot P_2 = 3\delta \underbrace{\left(\phi\left(p_{B,o}^{2*}\right) - V\right)}_{>0, \text{ by Lemma A1-(ii)}} \underbrace{\left[\left(p_{B,o}^{2*} - c\right)\left(\phi\left(p_{B,o}^{2*}\right) q\left(p_{B,o}^{2*}\right)\right)' - \phi\left(p_{B,o}^{2*}\right)\phi'\left(p_{B,o}^{2*}\right)\right]}_{<0, \text{ by Lemma A2-(i)}}$$
$$+ (9 - 4\delta)\left(q\left(p_{B,o}^{2*}\right) + 2\phi'\left(p_{B,o}^{2*}\right)\right) \underbrace{\left[\phi\left(p_{B,o}^{2*}\right)\left[3\pi\left(p_{B,o}^{2*}\right) - V - \phi\left(p_{B,o}^{2*}\right)\right] - V^2\right]}_{<0, \text{ by Lemma A2-(iii)}}$$
$$+ 3\delta\phi\left(p_{B,o}^{2*}\right) q\left(p_{B,o}^{2*}\right) \underbrace{\left(\pi\left(p_{B,o}^{2*}\right) - V\right)}_{\leq 0}$$
$$+ \delta \underbrace{\left(q\left(p_{B,o}^{2*}\right) - \phi'\left(p_{B,o}^{2*}\right)\right)}_{<0, \text{ by Lemma A1-(i)}} \underbrace{\left[\phi\left(p_{B,o}^{2*}\right)\left(\pi\left(p_{B,o}^{2*}\right) - V\right) + 2V\left(\phi\left(p_{B,o}^{2*}\right) - V\right)\right]}_{>0, \text{ by Lemma A2-(iv)}}.$$

From Lemma A2, $P_2 \geq 0$ for any $p_{B,o}^{2*} > c$. Which allows us to conclude that $\pi_{A,s}^* - \pi_{B,l}^* > 0$. Finally, let $t^* \equiv \min\{t_b, t_c, t_d, t_e\}$, so that (a)-(e) hold true for any $t \in (0, t^*)$.

Proof of Proposition 6. In order to show that firms choose short-term memberships in equilibrium, we have to show that for t small

(i) $\pi_{A,s}(t) > \pi_l(t)$ and
(ii) $\pi_s(t) > \pi_{B,l}(t)$.

(i) Let us show that, for t small, $\pi_{A,s}(t) - \pi_l(t) > 0$. From (A.40), we know that

$$\pi_{A,s}(t) = x^* F_A^* + \delta s_A\left(c, F_{A,o}^{2*}, c, F_{B,n}^{2*}\right) F_{A,o}^{2*} + \delta\left[s_B\left(p_{B,o}^2, c, F_{A,n}^{2*}\right) - x^*\right] F_{A,n}^{2*}, \tag{A.62}$$

and from (A.29)

$$\begin{aligned} \pi_l(t) = \frac{t}{2} + \frac{\delta t}{2} \cdot \frac{q(p_o^2)}{2\phi'(p_o^2) + q(p_o^2)} + \frac{\delta}{2} \cdot \frac{\phi'(p_o^2) + q(p_o^2)}{2\phi'(p_o^2) + q(p_o^2)}\left[v(c) - v(p_o^2) - \pi(p_o^2)\right] \\ + \delta\left[\pi(p_o^2)\left(\frac{1}{2} + \frac{v(p_o^2) - v(c)}{4t}\right) + \frac{(v(c) - v(p_o^2))^2}{8t}\right]. \end{aligned} \tag{A.63}$$

From (A.46), $p_{B,o}^{2*}$ is implicitly defined by

$$t = \frac{1}{3 - 2x^*}\left(2\phi\left(p_{B,o}^{2*}\right) + v(c) - v\left(p_{B,o}^{2*}\right)\right), \tag{A.64}$$

and, from (A.25), p_o^2 is implicitly defined by

$$t = \frac{1}{2}\left(\psi\left(p_o^2\right) + v(c)\right). \tag{A.65}$$

From (A.64) and (A.65) combined,

$$\psi\left(p_o^2\right) = \frac{2}{3-2x^*}\left(\psi\left(p_{B,o}^{2*}\right) + v\left(c\right)\right) - v\left(c\right). \tag{A.66}$$

It follows that p_o^2 can be seen as a function of $p_{B,o}^{2*}$, $p_o^2 = p_o^2\left(p_{B,o}^{2*}\right)$. Using (A.44), (A.45) and (A.47) into (A.62), it follows that there exists functions $N_{A,s}\left(p_{B,o}^{2*}\right)$ and $D_{A,s}\left(p_{B,o}^{2*}\right)$ such that

$$\pi_{A,s}\left(t\right) = \frac{N_{A,s}\left(p_{B,o}^{2*}\right)}{D_{A,s}\left(p_{B,o}^{2*}\right)}. \tag{A.67}$$

Similarly, from (A.63), there exists functions $N_l\left(p_o^2\right)$ and $D_l\left(p_o^2\right)$ such that

$$\pi_l\left(t\right) = \frac{N_l\left(p_o^2\right)}{D_l\left(p_o^2\right)}. \tag{A.68}$$

To show that $\pi_{A,s}(t) - \pi_l(t) > 0$, as $t \to 0$, we need to use L'hospital rule since $N_{A,s}\left(p_{B,o}^{2*}\right)$, $D_{A,s}\left(p_{B,o}^{2*}\right)$, $N_l\left(p_o^2\right)$ and $D_l\left(p_o^2\right)$ converge to 0 as $t \to 0$. Using the chain rule and (A.66), it follows that

$$\begin{aligned} &\frac{\partial}{\partial p_{B,o}^{2*}}\left[\pi_l\left(t\right)\right] = \frac{\partial}{\partial p_o^2}\left[\pi_l\left(t\right)\right] \cdot \frac{\partial p_o^2}{\partial p_{B,o}^{2*}}, \text{ and} \\ &\frac{\partial^2}{\partial\left(p_{B,o}^{2*}\right)^2}\left[\pi_l\left(t\right)\right] = \frac{\partial^2}{\partial\left(p_o^2\right)^2}\left[\pi_l\left(t\right)\right] \cdot \left(\frac{\partial p_o^2}{\partial p_{B,o}^{2*}}\right)^2 + \frac{\partial}{\partial p_o^2}\left[\pi_l\left(t\right)\right] \cdot \frac{\partial^2 p_o^2}{\partial\left(p_{B,o}^{2*}\right)^2}. \end{aligned} \tag{A.69}$$

Note that from (A.66) and the Implicit Function Theorem it follows that

$$\frac{\partial p_o^2}{\partial p_{B,o}^{2*}} = \frac{2}{\psi'\left(p_o^2\right)} \cdot \frac{\psi'\left(p_{B,o}^{2*}\right)\left(3-2x^*\right) + 2\frac{\partial x^*}{\partial p_{B,o}^{2*}}\left(\psi\left(p_{B,o}^{2*}\right) + v\left(c\right)\right)}{\left(3-2x^*\right)^2}. \tag{A.70}$$

Moreover, $\left.\frac{\partial p_o^2}{\partial p_{B,o}^{2*}}\right|_{t=0} = 1$. Also, from (A.70) and the Implicit Function Theorem

$$\begin{aligned} &\frac{1}{2}\left(3-2x^*\right)^3 \cdot \left\{\psi''\left(p_o^2\right)\left(\frac{\partial p_o^2}{\partial p_{B,o}^{2*}}\right)^2 + \psi'\left(p_o^2\right)\frac{\partial^2 p_o^2}{\partial\left(p_{B,o}^{2*}\right)^2}\right\} \\ &= \left(\psi''\left(p_{B,o}^{2*}\right)\left(3-2x^*\right) + 4\frac{\partial x^*}{\partial p_{B,o}^2}\psi'\left(p_{B,o}^{2*}\right)\right)\left(3-2x^*\right) \\ &+ \left(2\left(3-2x^*\right)\frac{\partial^2 x^*}{\partial\left(p_{B,o}^{2*}\right)^2} + 8\left(\frac{\partial x^*}{\partial p_{B,o}^{2*}}\right)^2\right)\left(\psi\left(p_{B,o}^{2*}\right) + v\left(c\right)\right). \end{aligned} \tag{A.71}$$

Moreover, as $t \to 0$

$$\left.\frac{\partial^2 p_o^2}{\partial \left(p_{B,o}^{2*}\right)^2}\right|_{t=0} = -\frac{5\delta q'(c)}{(27-11\delta)\, q(c)}.$$

Now, note that the functions $N_{A,s}\left(p_{B,o}^{2*}\right)$ and $D_{A,s}\left(p_{B,o}^{2*}\right)$ are such that

(A.72)
$$\begin{aligned}
\pi_{A,s}(t)\Big|_{t=0} &= 0, \\
\frac{\partial}{\partial p_{B,o}^{2*}}\left[\pi_{A,s}(t)\right]\Big|_{t=0} &= \frac{1}{12}(8\delta+9)\, q(c), \text{ and} \\
\frac{\partial^2}{\partial \left(p_{B,o}^{2*}\right)^2}\left[\pi_{A,s}(t)\right]\Big|_{t=0} &= \frac{\left(110\delta^2-12\delta-81\right) q'(c)}{132\delta-324}.
\end{aligned}$$

The functions $N_l\left(p_o^2\right)$ and $D_l\left(p_o^2\right)$ are such that

(A.73)
$$\begin{aligned}
\pi_l(t)\Big|_{t=0} &= 0, \\
\frac{\partial}{\partial p_o^2}\left[\pi_l(t)\right]\Big|_{t=0} &= \frac{1}{12}(8\delta+9)\, q(c), \\
\frac{\partial^2}{\partial \left(p_o^2\right)^2}\left[\pi_l(t)\right]\Big|_{t=0} &= \frac{1}{36}(28\delta+9)\, q'(c).
\end{aligned}$$

Finally, from (A.73), (A.70), (A.71) and (A.69),

(A.74)
$$\begin{aligned}
\frac{\partial}{\partial p_{B,o}^{2*}}\left[\pi_l(t)\right]\Big|_{t=0} &= \frac{1}{12}(8\delta+9)\, q(c), \text{ and} \\
\frac{\partial^2}{\partial \left(p_{B,o}^{2*}\right)^2}\left[\pi_l(t)\right]\Big|_{t=0} &= \frac{\left(428\delta^2-522\delta-243\right) q'(c)}{396\delta-972}.
\end{aligned}$$

Note that from (A.72) and (A.74) it follows that

$$\frac{\partial}{\partial p_{B,o}^{2*}}\left[\pi_{A,s}(t)-\pi_l(t)\right]\Big|_{t=0} = 0.$$

Using L'hospital rule again we find that

$$\frac{\partial^2}{\partial \left(p_{B,o}^{2*}\right)^2}\left[\pi_{A,s}(t)-\pi_l(t)\right]\Big|_{t=0} = -\frac{\delta(243-49\delta)\, q'(c)}{486-198\delta} > 0.$$

By continuity $\pi_{A,s}(t)-\pi_l(t) > 0$ for $t > 0$ small.

(ii) Let us show that, for t small, $\pi_s(t)-\pi_{B,l}(t) > 0$. From (A.41), we know that

(A.75)
$$\pi_{B,l}(t) = (1 - x^*(\alpha)) F_B^* + \delta \left(1 - s_B\left(p_{B,o}^{2*}, c, F_{A,n}^{2*}\right)\right) \pi\left(p_{B,o}^{2*}\right) + \delta \left[x^* - s_A\left(c, F_{A,o}^{2*}, c, F_{B,n}^{2*}\right)\right] F_{B,n}^{2*},$$

and, from (A.30),

$$\pi_s(t) = \frac{t}{18}(9 + 8\delta). \tag{A.76}$$

Using (A.44), (A.45) and (A.47) into (A.75), it is possible to show that there exists functions $N_{sl}^B\left(p_{B,o}^{2*}\right)$ and $D_{sl}^B\left(p_{B,o}^{2*}\right)$ such that

$$\pi_s(t) - \pi_{B,l}(t) = \frac{N_{sl}^B\left(p_{B,o}^{2*}\right)}{D_{sl}^B\left(p_{B,o}^{2*}\right)}$$

Note that $N_{sl}^B\left(p_{B,o}^{2*}\right)$ and $D_{sl}^B\left(p_{B,o}^{2*}\right)$ converge to 0 as $t \to 0$. Using L'hospital rule it follows that when $t \to 0$, $p_{B,o}^{2*} \to c$ and

$$\begin{aligned}
\pi_s(t) - \pi_{B,l}(t)\Big|_{t=0} &= 0, \\
\frac{\partial}{\partial p_{B,o}^{2*}}\left[\pi_s(t) - \pi_{B,l}^*(t)\right]\Big|_{t=0} &= 0, \text{ and} \\
\frac{\partial^2}{\partial \left(p_{B,o}^{2*}\right)^2}\left[\pi_s(t) - \pi_{B,l}^*(t)\right]\Big|_{t=0} &= -\frac{\delta(243 - 49\delta) q'(c)}{486 - 198\delta} > 0.
\end{aligned}$$

By continuity $\pi_s(t) - \pi_{B,l}(t) > 0$ for $t > 0$ small.

Corollary A1. *In the long-term contract for the unit price game:*

(i) $\frac{\partial x^*}{\partial F_A}, \frac{\partial p_{A,o}^2}{\partial F_A} < 0$ $\frac{\partial \underline{s}_A}{\partial F_A} > 0$.

(ii) $\frac{\partial \underline{s}_A}{\partial F_A^l}, \frac{\partial x^*}{\partial F_A^l} < 0$, $\frac{\partial p_{A,o}^2}{\partial F_A^l} > 0$.

(iii) $q(p_A)\left[\begin{array}{ccc} \frac{\partial x^*}{\partial F_A} & \frac{\partial \underline{s}_A}{\partial F_A} & \frac{\partial p_{A,o}^2}{\partial F_A}\end{array}\right]' = \left[\begin{array}{ccc} \frac{\partial x^*}{\partial p_A} & \frac{\partial \underline{s}_A}{\partial p_A} & \frac{\partial p_{A,o}^2}{\partial p_A}\end{array}\right]'$.

(iv) $q\left(p_A^l\right)(1+\delta)\left[\begin{array}{ccc} \frac{\partial x^*}{\partial F_A^l} & \frac{\partial \underline{s}_A}{\partial F_A^l} & \frac{\partial p_{A,o}^2}{\partial F_A^l}\end{array}\right]' = \left[\begin{array}{ccc} \frac{\partial x^*}{\partial p_A^l} & \frac{\partial \underline{s}_A}{\partial p_A^l} & \frac{\partial p_{A,o}^2}{\partial p_A^l}\end{array}\right]'$.

Proof of Corollary A1. Let $\psi\left(p_{A,o}^2\right) = 2\zeta\left(p_{A,o}^2\right) + v\left(p_{A,o}^2\right)$; thus,

$$f_1 = 2t(1 - 2\underline{s}_A) + t(2x^* - 1) - v(c) - \psi\left(p_{A,o}^2\right),$$

$$f_2 = -4tx^* + 2t + v(p_A) - v(p_B) + F_B - F_A + v\left(p_A^l\right)(1+\delta) - F_A^l - v\left(p_B^l\right)(1+\delta) - F_B^l,$$

and

$$f_3 = v\left(p_A^l\right)(1+\delta) - F_A^l - v\left(p_A\right) + F_A - \delta v\left(p_{A,o}^2\right).$$

Thus using the implicit function theorem,

$$\begin{bmatrix} \frac{\partial x^*}{\partial p_A} \\ \frac{\partial \underline{s}_A}{\partial p_A} \\ \frac{\partial p_{A,o}^2}{\partial p_A} \end{bmatrix} = -q\left(p_A\right) \begin{bmatrix} \frac{1}{4t} \\ \frac{1}{8t} - \frac{\psi'\left(p_{A,o}^2\right)}{4t\delta q\left(p_{A,o}^2\right)} \\ \frac{1}{\delta q\left(p_{A,o}^2\right)} \end{bmatrix}.$$

Note that

$$\begin{bmatrix} \frac{\partial x^*}{\partial F_A} \\ \frac{\partial \underline{s}_A}{\partial F_A} \\ \frac{\partial p_{A,o}^2}{\partial F_A} \end{bmatrix} q\left(p_A\right) = \begin{bmatrix} \frac{\partial x^*}{\partial p_A} \\ \frac{\partial \underline{s}_A}{\partial p_A} \\ \frac{\partial p_{A,o}^2}{\partial p_A} \end{bmatrix}.$$

Similarly,

$$\begin{bmatrix} \frac{\partial x^*}{\partial F_A^l} \\ \frac{\partial \underline{s}_A}{\partial F_A^l} \\ \frac{\partial p_{A,o}^2}{\partial F_A^l} \end{bmatrix} q(p_A^l)\left(1+\delta\right) = -q(p_A^l)\left(1+\delta\right) \begin{bmatrix} \frac{1}{4t} \\ \frac{1}{8t} + \frac{\psi'\left(p_{A,o}^2\right)}{4t\delta q\left(p_{A,o}^2\right)} \\ -\frac{1}{\delta q\left(p_{A,o}^2\right)} \end{bmatrix} = \begin{bmatrix} \frac{\partial x^*}{\partial p_A^l} \\ \frac{\partial \underline{s}_A}{\partial p_A^l} \\ \frac{\partial p_{A,o}^2}{\partial p_A^l} \end{bmatrix}.$$

Proof of Proposition 7. Let us show first that marginal-cost pricing is an equilibrium. The first-order condition after using the envelope theorem with respect to p_A is

$$\begin{aligned} \text{(A.77)} \qquad & \frac{\partial \underline{s}_A}{\partial p_A}\left(\pi\left(p_A^l\right) + F_A^l\right) + \left(\frac{\partial x^*\left(\alpha\right)}{\partial p_A} - \frac{\partial \underline{s}_A}{\partial p_A}\right)\left(\pi\left(p_A\right) + F_A\right) + \left(x^*\left(\alpha\right) - \underline{s}_A\right)\pi'\left(p_A\right) \\ & + \delta\left\{-\frac{\partial \underline{s}_A}{\partial p_A} + \frac{\partial s_A\left(p_{A,o}^2, p_{B,n}^2, F_{B,n}^2\right)}{\partial F_{B,n}^2}\frac{\partial F_{B,n}^2}{\partial p_A}\right\}\pi\left(p_{A,o}^2\right) \\ & + \delta\left[\underbrace{\frac{\partial s_B\left(p_{B,o}^2, p_{A,n}^2, F_{A,n}^2\right)}{\partial p_{B,o}^2}\frac{\partial p_{B,o}^2}{\partial p_A}}_{=0} - \frac{\partial s^*}{\partial p_A}\right]F_{A,n}^2 = 0, \end{aligned}$$

and for F_A,

$$\begin{aligned} \text{(A.78)} \qquad & \frac{\partial \underline{s}_A}{\partial F_A}\left(\pi\left(p_A^l\right) + F_A^l\right) + \left(\frac{\partial x^*\left(\alpha\right)}{\partial F_A} - \frac{\partial \underline{s}_A}{\partial F_A}\right)\left(\pi\left(p_A\right) + F_A\right) + \left(x^*\left(\alpha\right) - \underline{s}_A\right) \\ & + \delta\left\{-\frac{\partial \underline{s}_A}{\partial F_A} + \frac{\partial s_A\left(p_{A,o}^2, p_{B,n}^2, F_{B,n}^2\right)}{\partial F_{B,n}^2}\frac{\partial F_{B,n}^2}{\partial F_A}\right\}\pi\left(p_{A,o}^2\right) \end{aligned}$$

$$+\delta\left[\underbrace{\frac{\partial s_B\left(p_{B,o}^2,p_{A,n}^2,F_{A,n}^2\right)}{\partial p_{B,o}^2}\frac{\partial p_{B,o}^2}{\partial F_A}}_{=0}-\frac{\partial x^*}{\partial F_A}\right]F_{A,n}^2=0.$$

Similarly, for p_A^l,

$$\frac{\partial \underline{s}_A}{\partial p_A^l}\left(\pi\left(p_A^l\right)+F_A^l\right)+\underline{s}_A\pi'\left(p_A^l\right)+\left(\frac{\partial x^*(\alpha)}{\partial F_A^l}-\frac{\partial \underline{s}_A}{\partial F_A^l}\right)\left(\pi\left(p_A\right)+F_A\right)$$
$$+\delta\left\{-\frac{\partial \underline{s}_A}{\partial p_A^l}+\frac{\partial s_A\left(p_{A,o}^2,p_{B,n}^2,F_{B,n}^2\right)}{\partial F_{B,n}^2}\frac{\partial F_{B,n}^2}{\partial p_A^l}\right\}\pi\left(p_{A,o}^2\right)$$
$$+\delta\left[\underbrace{\frac{\partial s_B\left(p_{B,o}^2,p_{A,n}^2,F_{A,n}^2\right)}{\partial p_B^2}\frac{\partial p_{B,o}^2}{\partial p_A^l}}_{=0}-\frac{\partial x^*}{\partial p_A^l}\right]F_{A,n}^2=0, \tag{A.79}$$

and for F_A^l,

$$\frac{\partial \underline{s}_A}{\partial F_A^l}\left(\pi\left(p_A^l\right)+F_A^l\right)+\underline{s}_A+\left(\frac{\partial x^*(\alpha)}{\partial F_A^l}-\frac{\partial \underline{s}_A}{\partial F_A^l}\right)\left(\pi\left(p_A\right)+F_A\right)$$
$$+\delta\left\{-\frac{\partial \underline{s}_A}{\partial F_A^l}+\frac{\partial s_A\left(p_{A,o}^2,p_{B,n}^2,F_{B,n}^2\right)}{\partial F_{B,n}^2}\frac{\partial F_{B,n}^2}{\partial F_A^l}\right\}\pi\left(p_{A,o}^2\right)$$
$$+\delta\left[\underbrace{\frac{\partial s_B\left(p_{B,o}^2,p_{A,n}^2,F_{A,n}^2\right)}{\partial p_B^2}\frac{\partial p_{B,o}^2}{\partial F_A^l}}_{=0}-\frac{\partial s^*}{\partial F_A^l}\right]F_{A,n}^2=0. \tag{A.80}$$

Claim. Marginal-cost pricing is a symmetric equilibrium (e.g. $p_A=p_A^l=c$). Multiplying (A.78) by $q(p_A)$ and subtracting from (A.77), and from Corollary A1 and the fact that

$$\frac{\partial F_B^2}{\partial p_A}=q(p_A)\frac{\partial F_B^2}{\partial F_A},$$

we get that

$$(x^*(\alpha)-\underline{s}_A)q'(p_A)(p_A-c)=0. \tag{A.81}$$

Given $F_B,F_B^l\geq 0$ and $p_B,p_B^l\in\mathcal{P}$, $x^*(\alpha)-\underline{s}_A$ is strictly decreasing with respect to p_A for any $p_A\in\mathcal{P}$ (i.e., (B.4) implies that $\frac{\partial x^*(\alpha)}{\partial p_A}<0$) and from Corollary A1, we know that $\frac{\partial \underline{s}_A}{\partial p_A}>0$. Thus, if there exists $\tilde{p}_A\in\mathcal{P}$ such that $x^*(\tilde{\alpha})-\underline{s}_A=0$, then for any $p_A\geq\tilde{p}_A$, the profit is zero. Since $q'(p_A) < 0$, then equation (A.81) is positive for $p_A < c$ and negative for any $p_A \in (c, \tilde{p}_A(\tilde{\alpha}))$. Thus, (B.5) is single-peaked in p_A and reaches a unique maximum at $p_A = c$.

Analogously, multiplying (A.80) by $q(p_A^l)(1+\delta)$ and subtracting from equation (A.79), from Corollary A1 and the fact that

$$\frac{\partial F_B^2}{\partial p_A^l} = q(p_A^l)(1+\delta)\frac{\partial F_B^2}{\partial F_A^l},$$

we get that

$$\underline{s}_A \; q'(p_A^l) \; (p_A^l - c) = 0, \tag{A.82}$$

which allows us to conclude that (B.5) is single-peaked in p_A^l and reaches a unique maximum at $p_A^l = c$.

Now, since marginal cost pricing is a symmetric equilibrium (i.e. $p_A = p_A^l = c$), F_A and F_A^l are such that

$$\frac{\partial \underline{s}_A}{\partial F_A}F_A^l + \left(\frac{\partial x^*(\alpha)}{\partial F_A} - \frac{\partial \underline{s}_A}{\partial F_A}\right)F_A + \left(\frac{1}{2} - \underline{s}_A\right)$$
$$+\delta\left\{\frac{\partial s_A\left(p_{A,o}^2, p_{B,n}^2, F_{B,n}^2\right)}{\partial F_{B,n}^2}\frac{\partial F_{B,n}^2}{\partial F_A} - \frac{\partial \underline{s}_A}{\partial F_A}\right\}\pi\left(p_{A,o}^2\right) - \delta\frac{\partial x^*(\alpha)}{\partial F_A}F_{A,n}^2 = 0$$

and

$$\frac{\partial \underline{s}_A}{\partial F_A^l}F_A^l + \left(\frac{\partial x^*(\alpha)}{\partial F_A^l} - \frac{\partial \underline{s}_A}{\partial F_A^l}\right)F_A + \underline{s}_A$$
$$+\delta\left\{\frac{\partial s_A\left(p_{A,o}^2, p_{B,n}^2, F_{B,n}^2\right)}{\partial F_{B,n}^2}\frac{\partial F_{B,n}^2}{\partial F_A^l} - \frac{\partial s_A}{\partial F_A^l}\right\}\pi\left(p_{A,o}^2\right) - \delta\frac{\partial x^*(\alpha)}{\partial F_A^l}F_{A,n}^2 = 0,$$

respectively. Note that

$$\frac{\partial x^*(\alpha)}{\partial F_A} - \frac{\partial \underline{s}_A}{\partial F_A} = -\frac{1}{8t} - \frac{\psi'\left(p_{A,o}^2\right)}{4t\delta q\left(p_{A,o}^2\right)} < 0$$

and

$$\frac{\partial F_{B,n}^2}{\partial F_A} = -\frac{\delta}{4\delta} - \frac{2}{4\delta} = -\frac{(2+\delta)}{4\delta}.$$

Similarly, from (B.2),

$$\underline{s}_A = \frac{1}{2} - \frac{\phi\left(p_{A,o}^2\right)}{2t} - \frac{v(c) - v\left(p_{A,o}^2\right)}{4t}.$$

We then have the following system of equations:

$$\left(\frac{1}{8t}+\frac{\psi'\left(p^2\right)}{4t\delta q\left(p^2\right)}\right)F_A=\left(-\frac{1}{8t}+\frac{\psi'\left(p^2\right)}{4t\delta q\left(p^2\right)}\right)F_A^l+\frac{\phi\left(p^2\right)}{2t}+\frac{v\left(c\right)-v\left(p^2\right)}{4t}$$
$$-\pi\left(p^2\right)\frac{1}{4t}-\pi\left(p^2\right)\frac{\psi'\left(p^2\right)}{4tq\left(p^2\right)}+\delta\frac{\left(v\left(c\right)-v\left(p^2\right)\right)}{8t}, \tag{A.83}$$

$$\left(\frac{1}{8t}+\frac{\psi'\left(p^2\right)}{4t\delta q\left(p^2\right)}\right)F_A^l=\frac{1}{2}-\frac{\phi\left(p^2\right)}{2t}-\frac{v\left(c\right)-v\left(p^2\right)}{4t}+\left(-\frac{1}{8t}+\frac{\psi'\left(p^2\right)}{4t\delta q\left(p^2\right)}\right)F_A$$
$$+\frac{\psi'\left(p^2\right)}{4tq\left(p^2\right)}\pi\left(p^2\right)+\pi\left(p^2\right)\frac{1}{4t}+\delta\frac{\left(v\left(c\right)-v\left(p^2\right)\right)}{8t}=0, \tag{A.84}$$

which can be expressed as

$$AF_A=BF_A^l+C$$

and

$$AF_A^l=BF_A+G,$$

where $A=\left(\frac{1}{8t}+\frac{\psi'\left(p^2\right)}{4t\delta q\left(p^2\right)}\right)$, $B=\left(-\frac{1}{8t}+\frac{\psi'\left(p^2\right)}{4t\delta q\left(p^2\right)}\right)$, $C=\frac{\phi\left(p^2\right)}{2t}+\frac{v(c)-v\left(p^2\right)}{4t}-\pi\left(p^2\right)\frac{1}{4t}-\pi\left(p^2\right)\frac{\psi'\left(p^2\right)}{4tq\left(p^2\right)}+\delta\frac{\left(v(c)-v\left(p^2\right)\right)}{8t}$, $D=\left(\frac{1}{8t}+\frac{\psi'\left(p^2\right)}{4t\delta q\left(p^2\right)}\right)$, $G=\frac{1}{2}-\frac{\phi\left(p^2\right)}{2t}-\frac{v(c)-v\left(p^2\right)}{4t}+\frac{\psi'\left(p^2\right)}{4tq\left(p^2\right)}\pi\left(p^2\right)+\pi\left(p^2\right)\frac{1}{4t}+\delta\frac{\left(v(c)-v\left(p^2\right)\right)}{8t}$; thus,

$$\left(A+B\right)F_A^l=\left(B+A\right)F_A+G-C.$$

Note that

$$A+B=\frac{\psi'\left(p^2\right)}{2t\delta q\left(p^2\right)}>0$$

and

$$G-C=\frac{1}{2}-\frac{\phi\left(p^2\right)}{t}-\frac{v\left(c\right)-v\left(p^2\right)}{2t}+\frac{1}{2t}\left(\frac{\psi'\left(p^2\right)}{q\left(p^2\right)}+1\right)\pi\left(p^2\right)>0.$$

Therefore,

$$F_A^l=t+\frac{t\delta q\left(p^2\right)}{\psi'\left(p^2\right)}\left(2\underline{s}_A-\frac{1}{2}\right)+\frac{1}{2}\delta\left\{1+\frac{q\left(p^2\right)}{\psi'\left(p^2\right)}\right\}\pi\left(p^2\right)+\delta F_{A,n}^2. \tag{A.85}$$

Similarly, it follows that

$$F_A=t-t\left(2\underline{s}_A-\frac{1}{2}\right)\frac{\delta q\left(p^2\right)}{\psi'\left(p^2\right)}-\frac{1}{2}\delta\left\{1+\frac{q\left(p^2\right)}{\psi'\left(p^2\right)}\right\}\pi\left(p^2\right)+\delta F_{A,n}^2. \tag{A.86}$$

From (A.85) and (A.86), it follows that p^2 is implicitly defined by

$$\left(\phi'\left(p^2\right)+q\left(p^2\right)\right)\left(v\left(c\right)-v\left(p^2\right)-\pi\left(p^2\right)\right)=q\left(p^2\right)\left(\frac{t}{2}-\phi\left(p^2\right)\right). \tag{A.87}$$

Proof of Proposition 8.

(i) From the first-order conditions with respect to F_i for firms $i\in\{A,B\}$, note that

$$F_B^2-F_A^2=2tx^*-t-\left(v\left(p_A^2\right)-v\left(p_B^2\right)\right)+\frac{\pi\left(p_A^2\right)}{2}-\frac{\pi\left(p_B^2\right)}{2}. \tag{A.88}$$

Using (A.88) in the first-order conditions with respect to p_i^2 for $i\in\{A,B\}$, we get

(A.89)

$$t-\psi\left(p_A^2\right)\left(\frac{t}{2}-x^*t\right)-\frac{3\phi\left(p_A^2\right)}{2}+\frac{\pi\left(p_A^2\right)}{2}-\frac{\pi\left(p_B^2\right)}{2}-\psi\left(p_A^2\right)\left[\frac{v\left(p_A^2\right)-v\left(p_B^2\right)}{2}\right]+v\left(p_A^2\right)-v\left(p_B^2\right)=0$$

and

(A.90)

$$t-\psi\left(p_B^2\right)\left(tx^*-\frac{t}{2}\right)-\frac{3\phi\left(p_B^2\right)}{2}+\frac{\pi\left(p_B^2\right)}{2}-\frac{\pi\left(p_A^2\right)}{2}-\psi\left(p_B^2\right)\left[\frac{v\left(p_B^2\right)-v\left(p_A^2\right)}{2}\right]+v\left(p_B^2\right)-v\left(p_A^2\right)=0,$$

where $\psi\left(p\right)=\frac{q(p)}{\pi'(p)}$.

Claim: Marginal-cost pricing is not an equilibrium. Suppose that marginal-cost pricing is a Nash equilibrium for both firms; then

$$q\left(c\right)\left[\frac{t}{2}+x^*t\right]=0,$$

which is a contradiction.

Note that in a symmetric equilibrium, firms charge a single unit price defined by

$$t=\frac{3}{2}\phi\left(p^2\right)$$

and "subsidize" new consumers to switch with a negative membership fee, $F^2=-\frac{\pi\left(p^2\right)}{2}$;

(ii) The problem of the first period: let $\alpha=\left(p_A,F_A,p_B,F_B\right)$

$$x^*\left(\alpha\right)\left(\pi\left(p_A\right)+F_A\right)+\delta s_A\left(p_A^2,p_B^2,F_B^2\right)\pi\left(p_A^2\right)$$

$$+\delta\left[s_B\left(p_A^2,p_B^2,F_A^2\right)-x^*\left(\alpha\right)\right]\left(\pi\left(p_A^2\right)+F_A^2\right).$$

The first-order conditions are

$$[p_A] \qquad \frac{\partial x^*(\alpha)}{\partial p_A}\left(\pi\left(p_A\right)+F_A\right)+x^*(\alpha)\,\pi'\left(p_A\right)+$$

$$\delta\left\{\underbrace{s_A\left(p_A^2,p_B^2,F_B^2\right)\pi'\left(p_A^2\right)\frac{\partial p_A^2}{\partial x^*}+\frac{\partial s_A\left(p_A^2,p_B^2,F_B^2\right)}{\partial p_A^2}\pi\left(p_A^2\right)\frac{\partial p_A^2}{\partial x^*}}_{a}\right.$$

$$\left.+\frac{\partial s_A\left(p_A^2,p_B^2,F_B^2\right)}{\partial p_B^2}\pi\left(p_A^2\right)\frac{\partial p_B^2}{\partial x^*}+\frac{\partial s_A\left(p_A^2,p_B^2,F_B^2\right)}{\partial F_B^2}\pi\left(p_A^2\right)\frac{\partial F_B^2}{\partial x^*}\right\}\frac{\partial x^*(\alpha)}{\partial p_A}$$

$$+\delta\left\{\left[s_B\left(p_A^2,p_B^2,F_A^2\right)-x^*(\alpha)\right]\left(\underbrace{\pi'\left(p_A^2\right)\frac{\partial p_A^2}{\partial x^*}}_{b}+\underbrace{\frac{\partial F_A^2}{\partial x^*}}_{d}\right)\right\}\frac{\partial x^*(\alpha)}{\partial p_A}$$

$$+\frac{\partial x^*(\alpha)}{\partial p_A}\left[\underbrace{\frac{\partial s_B\left(p_A^2,p_B^2,F_A^2\right)}{\partial p_A^2}\frac{\partial p_A^2}{\partial x^*}}_{c}+\frac{\partial s_B\left(p_A^2,p_B^2,F_A^2\right)}{\partial p_B^2}\frac{\partial p_B^2}{\partial x^*}+\underbrace{\frac{\partial s_B\left(p_A^2,p_B^2,F_A^2\right)}{\partial F_A^2}\frac{\partial F_A^2}{\partial x^*}}_{e}-1\right]=0$$

$$\left(\pi\left(p_A^2\right)+F_A^2\right)$$

and

$$[F_A]: \qquad \frac{\partial x^*(\alpha)}{\partial F_A}\left(\pi\left(p_A\right)+F_A\right)+x^*(\alpha)+\frac{\partial x^*(\alpha)}{\partial F_A}\times$$

$$\delta\left\{\underbrace{s_A\left(p_A^2,p_B^2,F_B^2\right)\pi'\left(p_A^2\right)\frac{\partial p_A^2}{\partial x^*}+\frac{\partial s_A\left(p_A^2,p_B^2,F_B^2\right)}{\partial p_A^2}\pi\left(p_A^2\right)\frac{\partial p_A^2}{\partial x^*}}_{a}\right.$$

$$\left.+\frac{\partial s_A\left(p_A^2,p_B^2,F_B^2\right)}{\partial p_B^2}\pi\left(p_A^2\right)\frac{\partial p_B^2}{\partial x^*}+\frac{\partial s_A\left(p_A^2,p_B^2,F_B^2\right)}{\partial F_B^2}\pi\left(p_A^2\right)\frac{\partial F_B^2}{\partial x^*}\right\}$$

$$+\delta\left\{\left[s_B\left(p_A^2,p_B^2,F_A^2\right)-x^*(\alpha)\right]\left(\underbrace{\pi'\left(p_A^2\right)\frac{\partial p_A^2}{\partial x^*}}_{b}+\underbrace{\frac{\partial F_A^2}{\partial x^*}}_{d}\right)\right\}\frac{\partial x^*(\alpha)}{\partial F_A}$$

$$+\frac{\partial x^*(\alpha)}{\partial F_A}\left[\underbrace{\frac{\partial s_B\left(p_A^2,p_B^2,F_A^2\right)}{\partial p_A^2}\frac{\partial p_A^2}{\partial x^*}}_{c}+\frac{\partial s_B\left(p_A^2,p_B^2,F_A^2\right)}{\partial p_B^2}\frac{\partial p_B^2}{\partial x^*}+\underbrace{\frac{\partial s_B\left(p_A^2,p_B^2,F_A^2\right)}{\partial F_A^2}\frac{\partial F_A^2}{\partial x^*}}_{e}-1\right]=0$$

$$\cdot\left(\pi\left(p_A^2\right)+F_A^2\right).$$

Using envelope theorem

$$\pi\left(p_A\right)+F_A+x^*\left(\alpha\right)\pi'\left(p_A\right)\left(\frac{\partial x^*\left(\alpha\right)}{\partial p_A}\right)^{-1}$$

$$+\,\delta\left\{\frac{\partial s_A\left(p_A^2,p_B^2,F_B^2\right)}{\partial p_B^2}\frac{\partial p_B^2}{\partial x^*}+\frac{\partial s_A\left(p_A^2,p_B^2,F_B^2\right)}{\partial F_B^2}\frac{\partial F_B^2}{\partial x^*}\right\}\pi\left(p_A^2\right) \tag{A.91}$$

$$+\delta\left[\frac{\partial s_B\left(p_A^2,p_B^2,F_B^2\right)}{\partial p_B^2}\frac{\partial p_B^2}{\partial x^*}-1\right]\left(\pi\left(p_A^2\right)+F_A^2\right)=0.$$

Similarly, for $[F_A]$,

$$\pi\left(p_A\right)+F_A+x^*\left(\alpha\right)\left(\frac{\partial x^*\left(\alpha\right)}{\partial F_A}\right)^{-1} \tag{A.92}$$

$$+\delta\left\{\frac{\partial s_A\left(p_A^2,p_B^2,F_B^2\right)}{\partial p_B^2}\frac{\partial p_B^2}{\partial x^*}+\frac{\partial s_A\left(p_A^2,p_B^2,F_B^2\right)}{\partial F_B^2}\frac{\partial F_B^2}{\partial x^*}\right\}\pi\left(p_A^2\right)$$

$$+\delta\left[\frac{\partial s_B\left(p_A^2,p_B^2,F_A^2\right)}{\partial p_B^2}\frac{\partial p_B^2}{\partial x^*}-1\right]\left(\pi\left(p_A^2\right)+F_A^2\right)=0,$$

using symmetry, we have that in equilibrium

$$\frac{\partial x^*\left(\alpha\right)}{\partial p_A}=\frac{\partial x^*\left(\alpha\right)}{\partial F_A}q\left(p_A\right),$$

which implies that $p=c$ is the unique symmetric equilibrium with

$$F_A=-\frac{1}{2}\left(\frac{\partial x^*\left(\alpha\right)}{\partial F_A}\right)^{-1}+\delta\frac{\pi\left(p^2\right)q'\left(p^2\right)\left(p^2-c\right)p_B'\left(x^*\right)}{4t},$$

which is equal to

$$F_A=t-\delta\left[\frac{\pi'\left(p^2\right)}{2}-\frac{q\left(p^2\right)\left(p^2-c\right)q'\left(p^2\right)\left(p^2-c\right)}{4t}\right]p_B^{2\prime}\left(x^*\right)>0.$$

Similarly, note that using symmetry,

$$F^2=-\frac{\pi\left(p^2\right)}{2}\quad\text{and}$$

$$t=\frac{3}{2}\phi\left(p^2\right),$$

thus,

$$F_A = t + \delta \left[\underbrace{-\frac{3q'(p^2)\, q(p^2)\,(p^2 - c)}{6q(p^2)}}_{>0} - \frac{q(p^2)^2}{2q(p^2)} + \underbrace{\frac{q'(p^2)^2 (p^2 - c)^2}{6q(p^2)}}_{>0} \right] p_B^{2\prime}(x^*) > 0.$$

Proof of Corollary 4.

(i) From Proposition D.2 (see Appendix D), we know that $p^2_{LP,n}, p^2_{LP,o}$ are such that

$$t - v\left(p^2_{LP,n}\right) = \phi\left(p^2_{LP,o}\right) - v\left(p^2_{LP,o}\right) \tag{A.93}$$

and

$$-v\left(p^2_{LP,o}\right) = \phi\left(p^2_{LP,n}\right) - v\left(p^2_{LP,n}\right). \tag{A.94}$$

From Corollary D.1 we know that $p^2_{LP,n} < p^2_{LP,o}$. From (A.94), it follows that $p^2_{LP,n} > c$. Similarly, from Proposition 1, we know that

$$t - \frac{\left[v(c) + v\left(p^2_{2PT,o}\right)\right]}{2} = \phi\left(p^2_{2PT,o}\right) - v\left(p^2_{2PT,o}\right). \tag{A.95}$$

First note that as $t \to 0$, $p^2_{2PT,o}, p^2_{LP,n}, p^2_{LP,o} \to c$. Also, for the symmetric case, we have

$$\begin{bmatrix} \frac{\partial p^2_{LP,o}}{\partial t} \\ \frac{\partial p^2_{LP,n}}{\partial t} \end{bmatrix} = -\frac{1}{\zeta'\left(p^2_{LP,o}\right)\zeta'\left(p^2_{LP,n}\right) - q\left(p^2_{LP,n}\right)q\left(p^2_{LP,o}\right)} \begin{bmatrix} -\zeta'\left(p^2_{LP,n}\right) \\ -q\left(p^2_{LP,o}\right) \end{bmatrix}$$

and

$$\frac{\partial p^2_{2PT,o}}{\partial t} = \frac{1}{\zeta'\left(p^2_{2PT,o}\right) - \frac{q\left(p^2_{2PT,o}\right)}{2}}.$$

If $t = 0$,

$$\begin{bmatrix} \frac{\partial p^2_{LP,o}}{\partial t} \\ \frac{\partial p^2_{LP,n}}{\partial t} \end{bmatrix} = \begin{bmatrix} \frac{2}{3q(c)} \\ \frac{1}{3q(c)} \end{bmatrix}$$

and

$$\frac{\partial p^2_{2PT,o}}{\partial t} = \frac{2}{3q(c)}.$$

If $t > 0$,

(A.96)
$$\begin{bmatrix} \frac{\partial p^2_{LP,o}}{\partial t} \\ \frac{\partial p^2_{LP,n}}{\partial t} \end{bmatrix} = \frac{1}{\Omega\left(p^2_{LP,o}, p^2_{LP,n}\right)} \begin{bmatrix} \phi'\left(p^2_{LP,n}\right) + q\left(p^2_{LP,n}\right) \\ q\left(p^2_{LP,o}\right) \end{bmatrix},$$

where $\Omega \equiv \phi'\left(p^2_{LP,o}\right)\phi'\left(p^2_{LP,n}\right) + \phi'\left(p^2_{LP,o}\right) q\left(p^2_{LP,n}\right) + \phi'\left(p^2_{LP,n}\right) q\left(p^2_{LP,o}\right)$. Note that

(A.97)
$$\frac{1}{\frac{\phi'\left(p^2_{LP,o}\right)\phi'\left(p^2_{LP,n}\right)+\phi'\left(p^2_{LP,o}\right)q\left(p^2_{LP,n}\right)+\phi'\left(p^2_{LP,n}\right)q\left(p^2_{LP,o}\right)}{\phi\prime\left(p^2_{LP,n}\right)+q\left(p^2_{LP,n}\right)}}$$
$$= \frac{1}{\phi'\left(p^2_{LP,o}\right) + \frac{\phi'\left(p^2_{LP,n}\right)q\left(p^2_{LP,o}\right)}{\phi'\left(p^2_{LP,n}\right)+q\left(p^2_{LP,n}\right)}} < \frac{1}{\phi'\left(p^2_{LP,o}\right) + \frac{q\left(p^2_{LP,o}\right)}{2}}.$$

Similarly, note that for $t > 0$,

(A.98)
$$\frac{\partial p^2_{2PT,o}}{\partial t} = \frac{1}{\phi'\left(p^2_{2PT,o}\right) + \frac{q\left(p^2_{2PT,o}\right)}{2}}.$$

Note that (A.97) and (A.98) are equal at $p^2_{LP,o} = p^2_{2PT,o} = c$ and that there is a one-to-one mapping between t and $p^2_{LP,o}$ as well as between t and $p^2_{2PT,o}$. Thus, it follows from (A.96), (A.97), and (A.98) that

$$\frac{\partial p^2_{2PT,o}}{\partial t} > \frac{\partial p^2_{LP,o}}{\partial t}.$$

Thus, in equilibrium, we have that $p^2_{LP,n} < p^2_{LP,o} < p^2_{2PT,o}$.

(ii) From (A.93),

$$\frac{v\left(p^2_{LP,o}\right) - v\left(p^2_{LP,n}\right)}{2} = \frac{\phi\left(p^2_{LP,o}\right) - t}{2}.$$

Similarly, from (A.95),

$$\frac{v\left(p^2_{2PT,o}\right) - v\left(c\right)}{4} = \frac{\phi\left(p^2_{2PT,o}\right) - t}{2}.$$

From the first part of this proposition, we know that $p^2_{2PT,o} > p^2_{LP,o}$; thus, $s^A_{LP} < s^A_{2PT}$.

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ONLINE APPENDIX (Not for Publication)

Appendix B: Long-Term Contracts (for the Unit Price)

On firm's A turf, the problem of firm A in period 2 is

$$\max_{p_{A,o}^2} \left(s_A\left(p_{A,o}^2, p_{B,n}^2, F_{B,n}^2\right) - \underline{s}_A\right)\pi\left(p_{A,o}^2\right),$$

where $s_A\left(p_{A,o}^2, p_{B,n}^2, F_{B,n}^2\right) \equiv \frac{1}{2} + \frac{v\left(p_{A,o}^2\right) - v\left(p_{B,n}^2\right) + F_{B,n}^2}{2t}$ and $s_A\left(p_{A,o}^2, p_{B,n}^2, F_{B,n}^2\right) - \underline{s}_A$ is the market share of consumers of firm A in period 2 who bought from firm A in period 1 but did not buy long-term contract. On A's turf, firm B offers a unit price and a membership fee to the new consumers, analogous to our membership (benchmark) model. Following the reasoning in Proposition 1, it is straightforward to show that in any interior equilibrium, firm B offers a marginal-cost-based 2PT and extracts surplus through the membership fee (i.e., $p_{B,n}^2 = c$), and

$$F_{B,n}^2 = \frac{t\left(2x^* - 1\right) + v\left(c\right) - v\left(p_{A,o}^2\right)}{2}, \tag{B.1}$$

where $p_{A,o}^2$ is uniquely defined by

$$2t\left(1 - 2\underline{s}_A\right) + t\left(2x^* - 1\right) - v\left(c\right) = 2\zeta\left(p_{A,o}^2\right) + v\left(p_{A,o}^2\right). \tag{B.2}$$

Similarly, an interior equilibrium requires that $x^* \in [\underline{x}, \bar{x}]$, so that both firms poach a positive share from their rival's turf.

The incentive constraint for the long-term contract is such that

$$v\left(p_A^l\right)\left(1 + \delta\right) - F_A^l = v\left(p_A\right) - F_A + \delta v\left(p_{A,o}^2\right) \tag{B.3}$$

Consumers are indifferent between paying the membership fee F^l and paying a unit price p_A^l in both periods, or pay the regular membership fee and marginal price in period 1 and pay $p_{A,o}^2$ in period 2.

Using (B.1) and (B.3) and the analogous first-order conditions on B's turf, the type-x^* consumer is such that

$$\begin{aligned} x^* = {} & \frac{1}{2} + \frac{v\left(p_A\right) - v(p_B)}{4t} + \frac{F_B - F_A}{4t} \\ & + \frac{v\left(p_A^l\right)\left(1 + \delta\right) - F_A^l}{4t} - \frac{v\left(p_B^l\right)\left(1 + \delta\right) - F_B^l}{4t}, \end{aligned} \tag{B.4}$$

which depends only on the marginal prices and membership fees of the first period. Thus, using (B.2), (B.3), and (B.4), we can determine how first-period pricing decisions would affect second-period prices and margins (e.g., $\underline{s}_A$ and x^*). Corollary A1 in Appendix A shows that an increase in the membership fee charged to consumers who do not buy long-term contracts, F_A, increases the share of consumers who prefer long-term contracts (making the standard membership offer less attractive) and decreases the second-period prices charged to old customers. A similar intuition follows when the membership fee of the long-term contracts increases. Note that the effect of F_A on $\left(x^*, \underline{s}_A, p_{A,o}^2\right)$ is equal to the effect of p_A divided by $q\left(p_A\right)$, suggesting that the most efficient way to extract surplus in the first period is to use marginal-cost-based membership fees; that is, to set marginal prices equal to the marginal cost and extract surplus through the membership fee.

In the first period, the problem of firm A is

$$\text{(B.5)}\quad \begin{gathered} \max_{p_A, F_A, p_A, F_A^l} \underbrace{\underline{s}_A\left(\pi\left(p_A^l\right)+F_A^l\right)}_{(1)}+\underbrace{\left(x^*(\alpha)-\underline{s}_A\right)\left(\pi\left(p_A\right)+F_A\right)}_{(2)}+ \\ \underbrace{\delta\left(s_A\left(p_{A,o}^2, p_{B,n}^2, F_{B,n}^2\right)-\underline{s}_A\right)\pi\left(p_{A,o}^2\right)}_{(3)}+\underbrace{\delta\left[s_B\left(p_{B,o}^2, p_{A,n}^2, F_{A,n}^2\right)-x^*(\alpha)\right]F_{A,n}^2}_{(4)} \\ s.t.\ \text{(B.3) and (B.4)}. \end{gathered}$$

Note that firm A's overall objective function now depends on four terms: (1) is equal to the share of consumers who buy a long-term contract from firm A in period 1, $\underline{s}_A$, multiplied by the long-term contract fee and the monopoly profit function; (2) is equal to the share of consumers who buy the standard membership offered by firm A in the first period, $\underline{s}_A - x^*$, multiplied by the standard membership fee and the monopoly profit function; (3) is equal to the market share of customers who buy from firm A the standard membership contract in period 1, and buy again from firm A in period 2, $s_A(\cdot) - \underline{s}_A$, multiplied by the monopoly profit function;[49] and (4) the share of switchers, that is those who buy from firm B in period 1 the standard membership contract and then buy from firm A in period 2, $s_B - x^*$, multiplied by the membership fee and the monopoly profit function charged to new customers.

Note that from Corollary A1 in Appendix A and the Implicit Function Theorem, we can express the second-period prices, p_A^2, $F_{A,n}^2$, $p_{B,n}^2$ and $F_{B,n}$, and both margins x^* and $\underline{s}_A$ in terms of p_A, F_A, p_A^l and F_A^l and find a solution to problem (B.5).

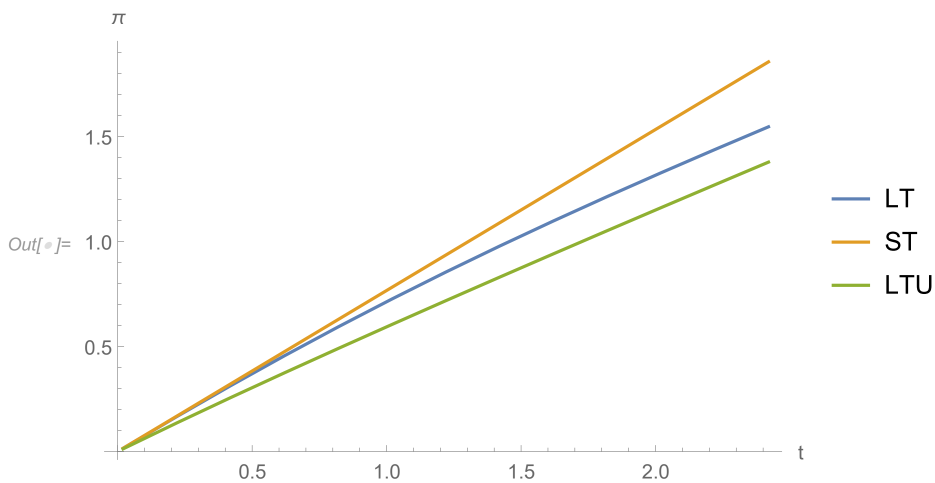
Simulation. Here we provide a numerical example to illustrate and compare the firm's profits when using memberships with long-term contracts, short-term memberships, and our benchmark model with long-term memberships. We use the following utility function:

[49] Remember that old customers do not have to pay the subscription fee again.

$u(p) = \frac{\alpha^{\frac{1}{\epsilon}} q^{1-\frac{1}{\epsilon}}}{1-\frac{1}{\epsilon}}$, which results in a constant elasticity demand curve (with elasticity equal to ϵ); i.e., $q(p) = \frac{\alpha}{p^{\epsilon}}$. In this case, $v(p) = \frac{1}{(\epsilon-1)} \frac{\alpha}{p^{\epsilon-1}}$. We assume the following parameters: $\alpha = 1$, $c = 0.3$, $\delta = 0.1$, $t = 0.25$ and $\epsilon = 2$. When $\epsilon = 2$, then $\phi(p) = \frac{p-c}{p(2c-p)}$.

When firms offer short-term memberships, they have more tools to extract consumer surplus in period 2 compared to the other two models. Note that long-term memberships with long-term contracts with the unit price reduce the captive demand of old customers in period 2, decreasing the overall profits. From the previous section, we know that as t increases, membership fees increase linearly. In contrast, the prices charged to the customers (when firms offer long-term memberships) depend on the curvature of the demand. Also, it becomes more challenging to switch to the rival firm in period 2, increasing the share of old customers who buy from the same firm they bought from in period 1.

Figure B1. Equilibrium Profits



Note: Figure B1 compares the equilibrium profits for our benchmark model (long-term membership) in blue, short-term membership in orange and long-term membership with long-term contracts in green, for different values of t.

Appendix C: Restricted Membership Model

In the second period, the problem of firm A is

$$\max_{p_A^2, \tilde{F}_A^2} \underbrace{s_A\left(p_A^2, p_B^2, F_{B,n}^2\right)}_{old} \pi\left(p_A^2\right) + \underbrace{\left(s_B\left(p_A^2, F_{A,n}^2, p_B^2\right) - x^*\right)}_{New}\left[\pi\left(p_A^2\right) + F_{A,n}^2\right]$$

where $s_A\left(p_A^2, p_B^2, F_{B,n}^2\right) \equiv \left\{\frac{1}{2} + \frac{v\left(p_A^2\right) - v\left(p_B^2\right) + F_{B,n}^2}{2t}\right\}$ is the market share of consumers on its own turf (share of old customers who buy again from firm A) and $s_B\left(p_A^2, F_{A,n}^2, p_B^2\right) \equiv \left\{\frac{1}{2} + \frac{v\left(p_A^2\right) - F_{A,n}^2 - v\left(p_B^2\right)}{2t}\right\}$ minus x^* is the market share of consumers who buy from firm A on B's turf (new customers). Note that firm A needs to find a price that maximizes the profits for both turfs, and charge a membership fee to the new customers. In equilibrium,

$$F_{i,n}^2 = \left(tx^* - \frac{t}{2}\right)\mathbf{1}_{[i]} - \left(\frac{v\left(p_j^2\right) - v\left(p_i^2\right)}{2}\right) - \frac{\pi\left(p_i^2\right)}{2} \tag{C.1}$$

where $\mathbf{1}_{[A]} = -1$, $\mathbf{1}_{[B]} = 1$, and p_A^2 and p_B^2 are defined by (A.89) and (A.90), respectively. Note that in this model, given that firms offered the same marginal price on both turfs, if firms set a positive membership fee, consumers will not have incentives to switch. Thus, in order to provide incentives for those consumers close to x^*, firms need to offer subsidies proportional to their demand. Moreover, (A.89) in Appendix A is not defined as $p_A^2 \to c$; thus, marginal-cost pricing is not an equilibrium. Similarly, (A.90) in Appendix A is not defined as $p_B^2 \to c$.

Working backward, we consider the optimal prices and consumption decisions in the first period to evaluate the overall impact of the information pricing that makes poaching possible. As in the previous models, first-period prices and membership fees will influence second-period pricing; firms thus take this into account in their first-period pricing decision.

In summary, in the membership fee model, when firms are not allowed to discriminate between new and old customers with their unit price (e.g., offering different monthly plans to old and new customers) in period 2, firms in equilibrium set a negative membership fee (subsidy) for new consumers, which is proportional to the equilibrium demand, and a single unit price to the entire market (both turfs). In period 1, firms offer cost-based membership fees.

Appendix D: Linear Pricing Game

In this section, we suppose that both firms use LP in both periods and are allowed to discriminate based on customers' purchase history, similar to Fudenberg and Tirole [11]. That is, each firm offers a price in the first period, p_i; and in the second period, each firm offers a price to its own past customers, $p^2_{i,o}$, as well as a price to those who purchased from its rival, $p^2_{i,n}$, for $i \in \{A, B\}$. A standard revealed-preference argument implies that at any pair of first-period prices there is a cutoff, x^*, such that all consumers with $x < x^*$ buy from firm A in the first period, and all consumers with $x > x^*$ buy from firm B in period 1. We named the space between 0 and x^* as the "turf" of firm A, and we consider the space to the right of x^* as the turf of firm B.

Second Period. On firm A's turf, the problem of firm A is

$$\max_{p^2_{A,o}} \; s_A\left(p^2_{A,o}, p^2_{B,n}\right)\pi\left(p^2_{A,o}\right), \tag{D.1}$$

where $s_A\left(p^2_{A,o}, p^2_{B,n}\right) \equiv \min\left\{x^*, \frac{1}{2} + \frac{v\left(p^2_{A,o}\right)-v\left(p^2_{B,n}\right)}{2t}\right\}$ is the share of consumers on A's turf who buy from firm A. Similarly, given that firm B will not capture the entire market on A's turf, the problem of firm B on A's turf is

$$\max_{p^2_{B,n}} \; \left\{x^* - s_A\left(p^2_{A,o}, p^2_{B,n}\right)\right\}\pi\left(p^2_{B,n}\right).$$

In an interior equilibrium, the two equations that defined $\left(p^2_{A,o}, p^2_{B,n}\right)$ are

$$t - v\left(p^2_{B,n}\right) = \zeta\left(p^2_{A,o}\right) \quad \text{and} \tag{D.2}$$

$$2tx^* - t - v\left(p^2_{A,o}\right) = \zeta\left(p^2_{B,n}\right), \tag{D.3}$$

where $\zeta(p) \equiv \phi(p) - v(p)$ and $\phi(p) \equiv \frac{\pi(p)q(p)}{\pi'(p)}$. The following proposition characterizes the equilibrium in period 2.

Proposition D.1. *Suppose (A1) and (A2) are satisfied.*

(i) In any equilibrium, if $x^* < 1$, $p^{2*}_{B,n} < p^{2*}_{A,o}$;

(ii) For $t > 0$, small, there exists $\underline{x} < \frac{1}{2}$ and $\bar{x} > \frac{1}{2}$ such that for $x^* \in [\underline{x}, \bar{x}]$, there exists a unique interior equilibrium, in which $\left(p^{2*}_{A,o}, p^{2*}_{B,n}\right)$ are defined by (D.2) and (D.3), and $s_A\left(p^{2*}_{A,o}, p^{2*}_{B,n}\right) \leq x^*$.

If $x^* = 1$, then from Armstrong and Vickers [2], we know that there exists a unique symmetric equilibrium with $p^{2*}_{i,o} = p^{2*}_{j,n}$ for $j \neq i$ and $i, j \in \{A, B\}$. That is, the turf of firm A in the second period would be the entire market, and the problem would be symmetric regarding the market share of both firms. Similarly, note that as $x^* \to \underline{x}$, there is no $p \in \mathcal{P}$ such that the market share for firm B would be positive; that is, at $p^2_{B,n} = c$ the indirect utility provided by firm B would not be enough to compensate the transportation cost. Note that the analysis is symmetric for firm B's turf; thus, for an interior equilibrium, we need that $x^* \in [\underline{x}, \bar{x}]$.[50] This interior equilibrium is unique with $p^{2*}_{j,n} < p^{2*}_{i,o}$ for $j \neq i$ and $i, j \in \{A, B\}$. That is, the poacher's price (and demand for the switchers) is strictly lower (higher) than the incumbent's price.[51] The following corollary characterizes the interior equilibrium in the second period.

Corollary D.1. *In any interior equilibrium,*

(i) $p^{2\prime}_{B,n}(x^*), p^{2\prime}_{A,o}(x^*) > 0$ *and* $p^{2\prime}_{B,o}(x^*), p^{2\prime}_{A,n}(x^*) < 0$.

(ii) *There exists* $\tilde{x}_B < \frac{1}{2}$ *such that* $\frac{\partial p^2_{B,n}}{\partial t} < (>) 0$ *if* $x^* > (<) \tilde{x}_B$.

(iii) $\frac{\partial p^2_{A,o}}{\partial t} > 0$.

First Period. Let's consider now the first-period pricing and consumers' decisions. Note that we assume that firms have no commitment power, so the prices and market share of the first period affect the second-period pricing strategy. Similarly, as we mentioned before, we assume that consumers are not myopic and they do anticipate the second-period pricing strategy of the firms. Thus, in any interior equilibrium, first-period prices imply that there is a consumer located at x^* who is indifferent between the two options, such as buying from firm A in period 1 and from firm B in period 2, and buying from firm B in period 1 and from firm A in period 2, which defines x^* as

$$x^* = \frac{1}{2} + \frac{v(p_A) - v(p_B) + \delta\left[v\left(p^2_{B,n}\right) - v\left(p^2_{A,n}\right)\right]}{2t(1-\delta)}. \tag{D.4}$$

Lemma D.1. *In equilibrium,* $\frac{\partial x^*}{\partial p_A} < 0$ *and* $\frac{\partial x^*}{\partial p_B} > 0$.

Note that $v\left(p^2_{B,n}\right) - v\left(p^2_{A,n}\right)$ is decreasing with respect to x^*.[52] Moreover, x^* is decreasing with respect to p_A and increasing with respect to p_B. Thus, for any p_A, p_B, x^* is uniquely defined. Let $\alpha = \left(p_A, p_B, p^2_{A,n}, p^2_{B,n}\right)$; then the problem of the firm A in the first period is

$$\max_{p_A} \quad x^*(\alpha)\,\pi(p_A) + \delta s_A\left(p^2_{A,o}, p^2_{B,n}\right)\pi\left(p^2_{A,o}\right)$$

[50] See the proof of Proposition D.1 for the analysis on B's turf.

[51] This result is similar to Fudenberg and Tirole [11] and consistent with Esteves and Reggiani [10], who also study a model, in which firms face a demand that can vary with the price level, using a different framework then the one used in this paper.

[52] Note that p^2_i and $p^2_{i,n}$ for $i \in \{A, B\}$ depends on x^*, which depends on p_A, p_B, i.e., $p^2_i = p^2_i(p_A, p_B)$. To simplify notation, we omit this for the rest of the appendix.

$$+\delta\left[s_B\left(p^2_{B,o},p^2_{A,n}\right)-x^*\left(\alpha\right)\right]\left(\pi\left(p^2_{A,n}\right)\right).$$

Remember that x^* is the share of consumers who buy from firm A in period 1; s_A is the share of consumers who buy from firm A in period 2 and who also buy from firm A in period 1, and $s_B\left(p^2_{B,o},p^2_{A,n}\right)-x^*\left(\alpha\right)$ is the share of consumers who buy from firm B in period 1 and buy from firm A in period 2 (the switchers). The problem for firm B is symmetric, and we exclude it here. The next proposition characterizes the equilibrium of the game.

Proposition D.2. *In any interior symmetric equilibrium,*

(i) $p>p^2_o$, where p is implicitly defined by (D.11).

(ii) $x^*=\frac{1}{2}$ and (p^2_n,p^2_o) are uniquely defined by (D.12) and (D.13).

(iii) $s_A=\frac{1}{2}+\frac{v\left(p^2_o\right)-v\left(p^2_n\right)}{2t}<\frac{1}{2}=x^*$ and $s_B=\frac{1}{2}+\frac{v\left(p^2_n\right)-v\left(p^2_o\right)}{2t}>\frac{1}{2}$.

Note that in equilibrium, the prices in the first period are higher than the prices in the second period. Intuitively, there are two effects: first, the market in period 2 is divided into two turfs, which makes firms compete more aggressively. Moreover, in order to attract new consumers, firms need to offer attractive prices that compensate for the higher transportation cost of the switchers. Similarly, note that in this general model, the share of switcher, $\frac{v\left(p^2_n\right)-v\left(p^2_o\right)}{t}$, depends on t and is different from 1/3, which contrasts with previous literature (e.g., Fudenberg and Tirole [11]).

Proofs of Appendix D

Proof of Proposition D.1.

(i) Note that from the first-order conditions,

$$t-v\left(p^2_{B,n}\right)=\zeta\left(p^2_{A,o}\right) \tag{D.5}$$

and

$$2tx^*-t-v\left(p^2_{A,o}\right)=\zeta\left(p^2_{B,n}\right). \tag{D.6}$$

Let's first assume that $x^*=1$; then

$$t-v\left(p^2_{B,n}\right)=\zeta\left(p^2_{A,o}\right)$$

and

$$t-v\left(p^2_{A,o}\right)=\zeta\left(p^2_{B,n}\right).$$

From Armstrong and Vickers [2], we know that there exists a unique symmetric equilibrium with $p^{2*}_{i,o}=p^{2*}_{j,n}$ for $i,j\in\{A,B\}$. Similarly, $s_A\left(p^{2*}_{A,o},p^{2*}_{B,n}\right)=\frac{1}{2}<x^*$.

Note that in any equilibrium, $p^{2*}_{A,o} > \tilde{p}^{2*}_{B,n}$ for $x^* < 1$. By contradiction, suppose that $p^{2*}_{A,o} \leq p^{2*}_{B,n}$. Then, from (D.5) and (D.6),

$$2tx^* - t - v\left(p^{2*}_{A,o}\right) = \zeta\left(p^{2*}_{B,n}\right) \geq \zeta\left(p^{2*}_{A,o}\right) = t - v\left(p^{2*}_{B,n}\right);$$

thus,

$$2t\left(x^* - 1\right) \geq v\left(p^{2*}_{A,o}\right) - v\left(p^{2*}_{B,n}\right) \geq 0,$$

which is a contradiction.

(ii) For $x^* = 1$, from Armstrong and Vickers [2], we know that there exists a unique symmetric equilibrium in which $p^{2*}_{i,o}$ is such that $\phi\left(p^{2*}_{i,o}\right) = t$. Note that for $x^* < 1$, as $x^* \to 0$, the intercept of $\xi^B\left(p^2_{A,o}\right)$ tends to $\zeta^{-1}\left(-t - v\left(c\right)\right)$, where $\xi^B\left(p^2_{A,o}\right)$ is such that $\left(p^2_{A,o}, \xi^B\left(p^2_{A,o}\right)\right)$ satisfies (D.6), but the slope remains constant. Similarly, note that the intercept and the slope of $\xi^A\left(p^2_{B,n}\right)$ remain constant where $\xi^A\left(p^2_{B,n}\right)$ is such that $\left(p^2_{B,n}, \xi^A\left(p^2_{B,n}\right)\right)$ satisfies (D.5). Thus, there exists $\underline{\mathrm{x}}$ such that for $x^* > \underline{\mathrm{x}}$ there exists a unique interior equilibrium.

Formally, using (D.5) in (D.6),

$$\left(2x^* - 1\right)\left[\zeta\left(p^2_{A,o}\right) + v\left(p^2_{B,n}\right)\right] - v\left(p^2_{A,o}\right) = \zeta\left(p^2_{B,n}\right),$$

note that as $x^* \to 0$, and the fact that in any equilibrium $p^{2*}_{A,o} > p^{2*}_{B,n}$, we conclude that there exists $\underline{\mathrm{x}}$ such that for $x < \underline{\mathrm{x}}$, there is not an interior equilibrium. Now let's show that there exists a unique interior equilibrium for $x > \underline{\mathrm{x}}$. From the first-order condition of firm A,

$$\text{(D.7)} \qquad \frac{\partial \Pi\left(p^2_{A,o}\right)}{\partial p^2_{A,o}} = \frac{\pi'\left(p^2_{A,o}\right)}{2t}\left\{t + v\left(p^2_{A,o}\right) - v\left(p^2_{B,n}\right) - \phi\left(p^2_{A,o}\right)\right\}.$$

Note that $\left\{t + v\left(p^2_{A,o}\right) - v\left(p^2_{B,n}\right) - \phi\left(p^2_{A,o}\right)\right\}$ is strictly decreasing with respect to $p^2_{A,o}$; thus, (D.1) is single-peaked with respect to $p^2_{A,o}$. Similarly, the problem of firm B is

$$\frac{\partial \Pi\left(p^2_{B,n}\right)}{\partial p^2_{B,n}} = \left\{x^* - \frac{1}{2} - \frac{v\left(p^2_{A,o}\right) - v\left(p^2_{B,n}\right)}{2t}\right\}\pi'\left(p^2_{B,n}\right) - \frac{\pi\left(p^2_{B,o}\right)q\left(p^2_{B,o}\right)}{2t}.$$

In equilibrium,

$$t\left(2x^* - 1\right) - \left(v\left(p^2_{A,o}\right) - v\left(p^2_{B,n}\right)\right) - \phi\left(p^2_{B,n}\right) = 0.$$

Note that

$$\frac{\partial \xi^B\left(p^2_{A,o}\right)}{\partial p^2_{A,o}} = \frac{q\left(p^2_{A,o}\right)}{q\left(p^2_{B,n}\right) + \phi'\left(p^2_{B,n}\right)}.$$

Thus, (D.7) can be expressed in terms of $p^2_{A,o}$:

$$\frac{\partial \Pi\left(p_A^2\right)}{\partial p_A^2}=\frac{\pi'\left(p_A^2\right)}{2t}\left\{t+v\left(p_A^2\right)-v\left(\xi^B\left(p_{A,o}^2\right)\right)-\phi\left(p_{A,o}^2\right)\right\}. \tag{D.8}$$

Note that as $p_{A,o}^2 \to c$, the right-hand side of (D.8) tends to a positive value, as $\xi^B(c)>c$. Similarly, as $p_{A,o}^2 \to p_A^m$, the right-hand side of (D.8) tends to a negative value. By continuity, a solution exists. Moreover, note that

$$t+v\left(p_{A,o}^2\right)-v\left(\xi^B\left(p_{A,o}^2\right)\right)-\phi\left(p_{A,o}^2\right),$$

is strictly decreasing with respect to $p_{A,o}^2$; thus the equilibrium is unique.

Proof of Corollary D.1.
(i) From the implicit function theorem,

$$\begin{bmatrix} \frac{\partial p_{A,o}^2}{\partial x^*} \\ \frac{\partial p_{B,n}^2}{\partial x^*} \end{bmatrix} = -\begin{bmatrix} -\zeta'\left(p_{A,o}^2\right) & q\left(p_{B,n}^2\right) \\ q\left(p_{A,o}^2\right) & -\zeta'\left(p_{B,n}^2\right) \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 2t \end{bmatrix}.$$

Note that

$$\begin{bmatrix} -\zeta'\left(p_{A,o}^2\right) & q\left(p_{B,n}^2\right) \\ q\left(p_{A,o}^2\right) & -\zeta'\left(p_{B,n}^2\right) \end{bmatrix}^{-1} = \frac{1}{\zeta'\left(p_{A,o}^2\right)\zeta'\left(p_{B,n}^2\right)-q\left(p_{A,o}^2\right)q\left(p_{B,n}^2\right)}\begin{bmatrix} -\zeta'\left(p_{B,n}^2\right) & -q\left(p_{B,n}^2\right) \\ -q\left(p_{A,o}^2\right) & -\zeta'\left(p_{A,o}^2\right) \end{bmatrix},$$

thus we have

$$\begin{bmatrix} \frac{\partial p_{A,o}^2}{\partial x^*} \\ \frac{\partial p_{B,n}^2}{\partial x^*} \end{bmatrix} = \begin{bmatrix} \frac{2tq\left(p_{B,n}^2\right)}{\zeta'\left(p_{A,o}^2\right)\zeta'\left(p_{B,n}^2\right)-q\left(p_{A,o}^2\right)q\left(p_{B,n}^2\right)} \\ \frac{2t\zeta'\left(p_{A,o}^2\right)}{\zeta'\left(p_{A,o}^2\right)\zeta'\left(p_{B,n}^2\right)-q\left(p_{A,o}^2\right)q\left(p_{B,n}^2\right)} \end{bmatrix}.$$

Note that $\zeta'\left(p_{A,o}^2\right)\zeta'\left(p_{B,n}^2\right)-q\left(p_{A,o}^2\right)q\left(p_{B,n}^2\right)>0$; thus $p_{B,n}^{2\prime}(x^*)>0$ and $p_{A,o}^{2\prime}(x^*)>0$.

(ii) From the implicit function theorem,

$$\begin{bmatrix} \frac{\partial p_{A,o}^2}{\partial t} \\ \frac{\partial p_{B,n}^2}{\partial t} \end{bmatrix} = -\frac{1}{\zeta'\left(p_{A,o}^2\right)\zeta'\left(p_{B,n}^2\right)-q\left(p_{A,o}^2\right)q\left(p_{B,n}^2\right)}\begin{bmatrix} -\phi'\left(p_{B,n}^2\right)-2x^*q\left(p_{B,n}^2\right) \\ -\left(2x^*-1\right)\phi'\left(p_{A,o}^2\right)-2x^*q\left(p_{A,o}^2\right) \end{bmatrix}.$$

Thus, we conclude that there exists $\tilde{x}_B$ such that $\frac{\partial p_{B,n}^2}{\partial t}>(<)0$ if $x^*>(<)\tilde{x}_B$.
(iii) Similarly, it follows that $\frac{\partial p_{A,o}^2}{\partial t}>0$.

Proof of Lemma D.1. We know that x^* must be indifferent between the two options; thus, from (D.4), it follows

$$\text{(D.9)} \qquad \frac{\partial x^*}{\partial p_A} = \frac{-q\left(p_A\right)}{\left\{2t\left(1-\delta\right)+\delta q\left(p_{B,n}^2\right)p_{B,n}^{2\prime}\left(x^*\right)-\delta q\left(p_{A,n}^2\right)p_{A,n}^{2\prime}\left(x^*\right)\right\}}$$

and

$$\text{(D.10)} \qquad \frac{\partial x^*}{\partial p_B} = \frac{q\left(p_B\right)}{\left\{2t\left(1-\delta\right)+\delta q\left(p_{B,n}^2\right)p_{B,n}^{2\prime}\left(x^*\right)-\delta q\left(p_{A,n}^2\right)p_{A,n}^{2\prime}\left(x^*\right)\right\}}.$$

Proof of Proposition D.2.

(i) From the first-order condition, after using the envelope theorem,

$$[p_A]: \quad \frac{\partial x^*\left(\alpha\right)}{\partial p_A}\pi\left(p_A\right)+x^*\left(\alpha\right)\pi'\left(p_A\right)+\delta\left\{\pi\left(p_{A,o}^2\right)\frac{\partial s_A\left(p_{A,o}^2,p_{B,n}^2\right)}{\partial p_{B,n}^2}\frac{\partial p_{B,n}^2}{\partial x^*}\right\}\frac{\partial x^*\left(\alpha\right)}{\partial p_A}$$

$$+\delta\left\{\pi\left(p_{A,n}^2\right)\frac{\partial s_B\left(p_{B,o}^2,p_{A,n}^2\right)}{\partial p_{B,o}^2}\frac{\partial p_{B,o}^2}{\partial x^*}-\pi\left(p_{A,n}^2\right)\right\}\frac{\partial x^*\left(\alpha\right)}{\partial p_A}=0.$$

Using symmetry (e.g. $p_{A,o}^2 = p_{B,o}^2$, $p_{A,n}^2 = p_{B,n}^2$, $p_A = p_B$ and $x^* = \frac{1}{2}$),

$$\text{(D.11)} \qquad \pi\left(p\right)+\frac{1}{2}\pi'\left(p\right)\left(\frac{\partial x^*\left(\alpha\right)}{\partial p_A}\right)^{-1}+\delta\frac{\pi\left(p_o^2\right)q\left(p_n^2\right)}{2t}\frac{\partial p_{B,n}^2}{\partial x^*}$$
$$+\delta\frac{\pi\left(p_n^2\right)q\left(p_o^2\right)}{2t}\frac{\partial p_{B,o}^2}{\partial x^*}-\delta\pi\left(p_n^2\right)=0.$$

Note that in any symmetric equilibrium $p_{A,n}^{2\prime}\left(x^*\right) = -p_{B,n}^{2\prime}\left(x^*\right)$, and $\left|p_{B,n}^{2\prime}\left(x^*\right)\right| > \left|p_{B,o}^{2\prime}\left(x^*\right)\right|$. Existence follows by the fact that as $p \to p_o^2$ the left-hand side of (D.11) is negative, and as $p \to p^m$ the left-hand side is positive. Thus, in equilibrium, $p > p^2$.

(ii) In any interior symmetric equilibrium, $x^* = \frac{1}{2}$, and from (D.5) and (D.6), it follows that (p_n^2, p_o^2) are uniquely defined by

$$\text{(D.12)} \qquad t - v\left(p_n^2\right) = \zeta\left(p_o^2\right)$$

and

$$\text{(D.13)} \qquad -v\left(p_o^2\right) = \zeta\left(p_n^2\right).$$